Computational aspects of reasoning about action

Jérôme Lang (IRIT, Toulouse)

Ontic vs. *epistemic* actions

- *Purely ontic (physical) actions*
  - may change the state of the world
  - do not bring feedback to the agent
    (what you foresee is what you get)

- *Purely epistemic actions (sensing actions)*
  - meant to provide new information
  - do not change the state of the world
    (only the agents beliefs)

binary tests : $\beta = \text{test}(\varphi)$ returns the truth value of φ

Ontic **vs.** ***epistemic*** **actions**

Complex actions have both ontic and epistemic effects (changes in the world + feedback)

Example: toss a coin (and observe the outcome)

However: any complex action can be written as a purely ontic action followed by a purely epistemic action

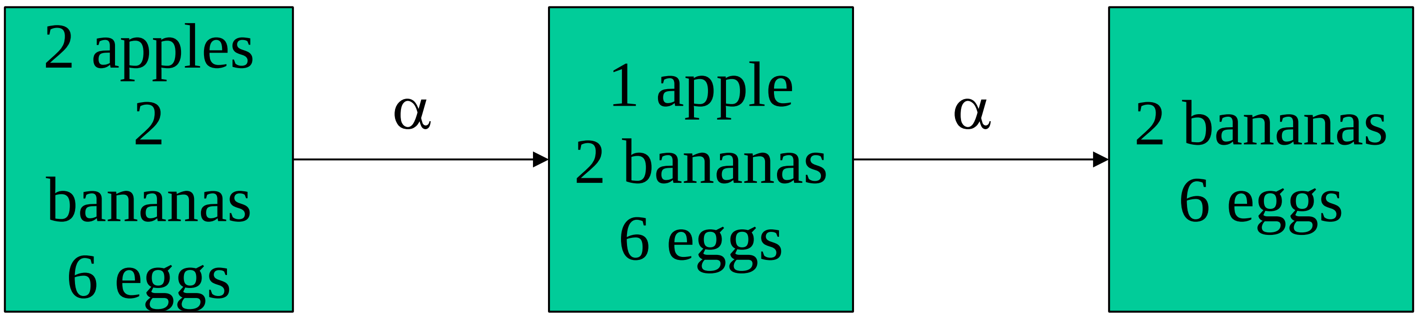
$$\alpha \equiv \alpha_E \circ \alpha_O$$

⇨ Without loss of generality: partition between purely ontic and purely epistemic actions

Ontic (physical) actions

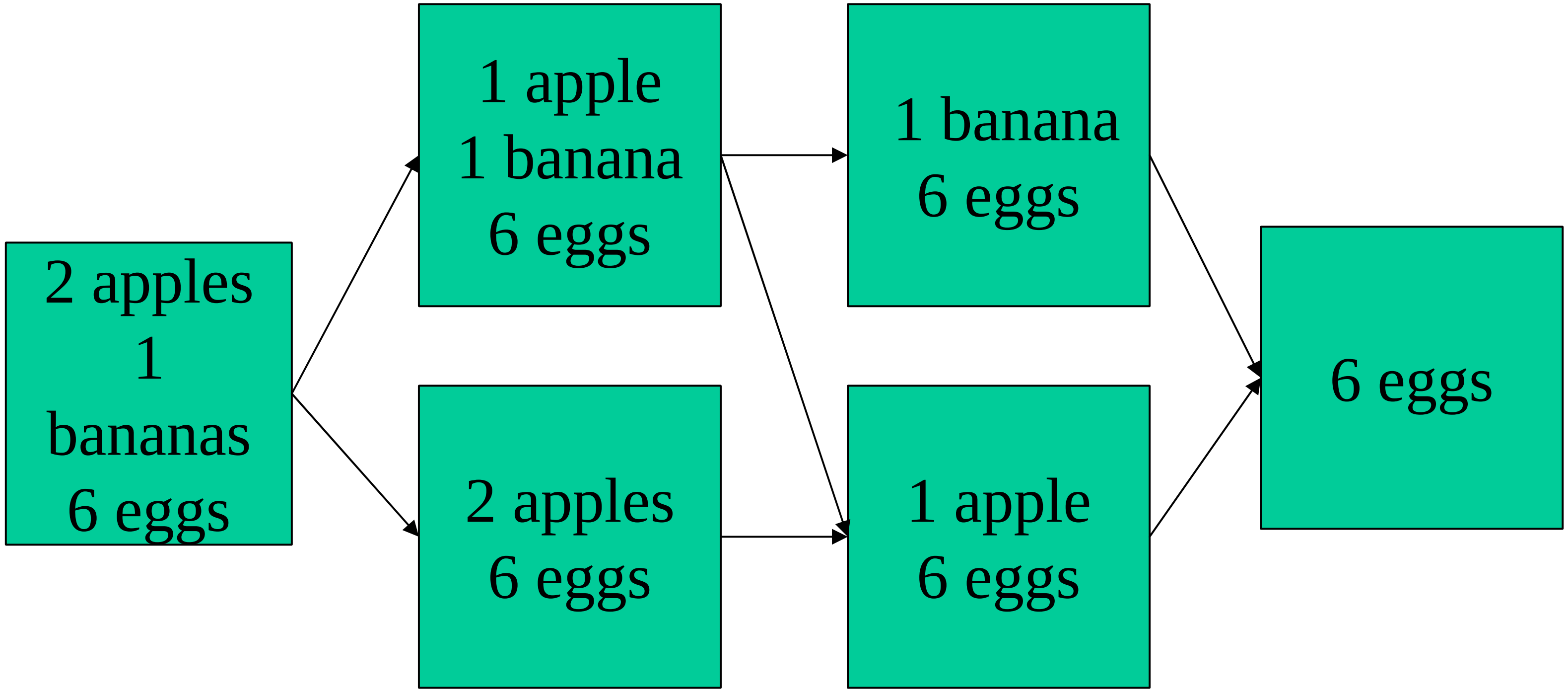
meant to change the state of the world

(physical) action = transition system = automaton



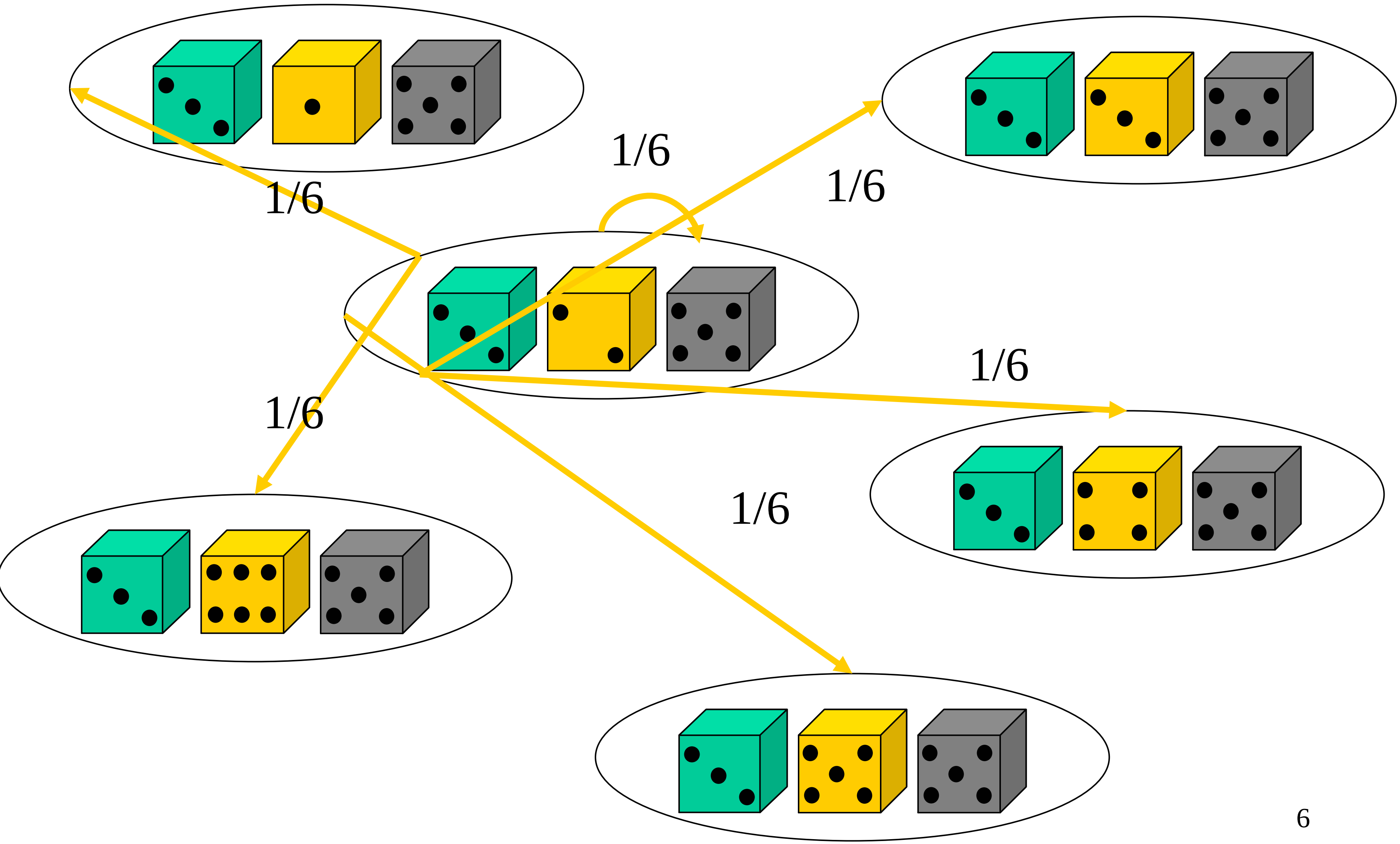
α = eat an apple

deterministic action = deterministic automaton



β = eat a fruit

nondeterministic action = nondeterministic automaton *stochastic action = stochastic automatpn*



combinatorial explosion

n fruits, *maximum p items of each* : $(p+1)^n$ possible states

+ temporal aspect $(p+1)^{n.T}$ possible **trajectories**

+ incomplete knowledge : $2^{(p+1)^n}-1$ possible **belief states**

(in the simplest uncertainty model)

how can action effects be represented in a concise way?

«Frame problem» : avoid specifying the properties of the world outside the scope of the action

« eat an apple » does not modify the number of bananas

Ramification problem: avoid specifying indirect effects indirects

empty-fridge = (0 apple and 0 banana)
« eat an apple» may influence « empty-fridge »

Compact expression of ontic action effects

Actions representation in **propositional logic**

$FL = \{a,b,c,d,\ldots\}$ finite set of *fluents* (propositional variables)

$S = 2^{FL}$ set of states

Notation : $s = \{a, \neg b, \neg c, d\}$ rather than $\{a,d\}$

literals

l true in $s \Leftrightarrow l \in s \Leftrightarrow s \models l$

l false in $s \Leftrightarrow \neg l \in s \Leftrightarrow s \models \neg l$

Compact expression of ontic action effects

An elementary action language: STRIPS

$\mathrm{eff}\,(\alpha) = \{\alpha$ **causes** $l_i\,,\ i = 1\,..\,q\,\}$ (l_i literals)

$\approx \{l_i\,,\ i = 1\,..\,q\,\}$

$$s \Diamond \mathrm{eff}\,(\alpha) = (\, s \setminus \{l \mid \neg\, l \in \mathrm{eff}\,(\alpha)\,\}\,) \cup \mathrm{eff}\,(\alpha)$$

$$\text{we have: } l \in (\, s \Diamond \mathrm{eff}\,(\alpha)) \text{ iff } \begin{cases} l \in \mathrm{eff}\,(\alpha) \\ \text{or} \\ l \in s \text{ et } \neg\, l \notin \mathrm{eff}\,(\alpha) \end{cases}$$

Example : $\mathrm{eff}\,(\alpha) = \{\alpha$ **causes** $\neg\, b,\ \alpha$ **causes** $\neg\, d\} \approx \{\neg\, b, \neg\, d\}$

$\{a, \neg b, \neg c,\ d\} \Diamond \mathrm{eff}\,(\alpha) = \{a, \neg b, \neg c, \neg\, d\}$

Compact expression of ontic action effects

An elementary action representation language : **STRIPS**

Why elementary?

1. no conditional effects
2. no nondeterminism
3. no causal relations between fluents
4. no concurrent actions

Compact expression of ontic action effects

1. conditional effects

$\text{eff}(\alpha) = \{$ **if** pre_i **then** α **causes** l_i , $i = 1 .. q \}$

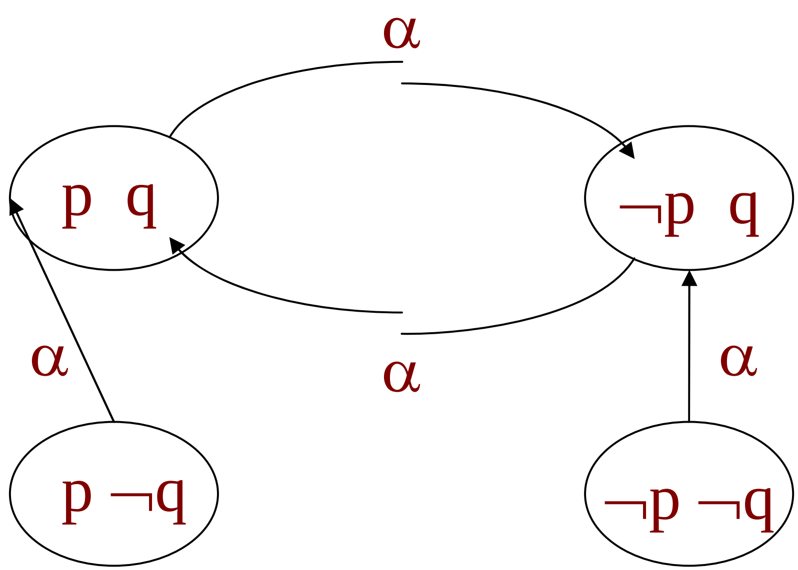
pre_i propositional fornula; l_i literal

$\text{filter}(s, \alpha) = \{ l_i \mid s \models \text{pre}_i \}$ assumed consistent

$s \blacklozenge \text{eff}(\alpha) = s \lozenge \text{filter}(s, \alpha)$

Compact expression of ontic action effects

eff (α) = { **if** $p \wedge q$ **then** α **causes** $\neg p$,
if $p \wedge q$ **then** α **causes** p,
(**if** T **then**) α **causes** q
}



Compact expression of ontic action effects

« Compilation » of an action

$$FL = \{a, b, \dots\} \quad \begin{cases} FL_t = \{a_t, b_t, \dots\} \\ FL_{t+1} = \{a_{t+1}, b_{t+1}, \dots\} \end{cases}$$

For all $f \in FL$: $\Gamma^+(f) = \bigvee \{pre_i \mid f = l_i\}$

$$\Gamma^-(f) = \Gamma^+(\neg f)$$

$$\boxed{f_{t+1} \equiv \Gamma^+(f)_t \vee (f_t \wedge \neg \Gamma^-(f)_t)}$$

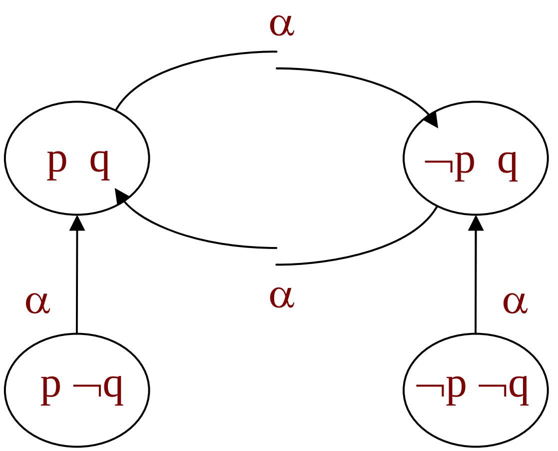
f is true at time t+1

if there is an applicable effect making it true

or f was true at time t and no applicable effect makes it false

Compact expression of ontic action effects

eff (α) = { **if** $p \wedge q$ **then** α **causes** $\neg p$,
if $\neg p \wedge q$ **then** α **causes** p,
(**if** $\top$ **then**) α **causes** q
}



$\Gamma^{+}(p) = \neg p \wedge q$

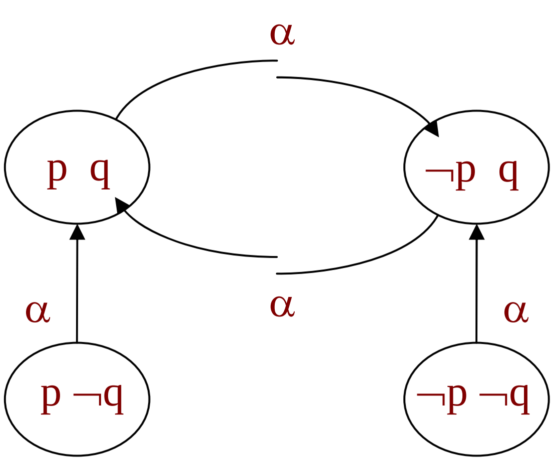
$\Gamma^{-}(p) = \Gamma^{+}(\neg p) = p \wedge q$

$\Gamma^{+}(q) = \top$

$\Gamma^{-}(q) = \bot$

Compact expression of ontic action effects

eff (α) = { **if** $p \wedge q$ **then** α **causes** $\neg p$,
if $\neg p \wedge q$ **then** α **causes** p,
(**if** $\top$ **then**) α **causes** q
}



$$\Gamma^{+}(p) = \neg p \wedge q$$

$$\Gamma^{-}(p) = \Gamma^{+}(\neg p) = p \wedge q$$

$$\Gamma^{+}(q) = \top$$

$$\Gamma^{-}(q) = \bot$$

$$\Sigma_{\alpha} : \boxed{\begin{array}{l} p_{t+1} \equiv ((\neg p_t \wedge q_t) \vee (p_t \wedge \neg (p_t \wedge q_t)) \\ q_{t+1} \equiv \top \end{array}}$$

Compact expression of ontic action effects

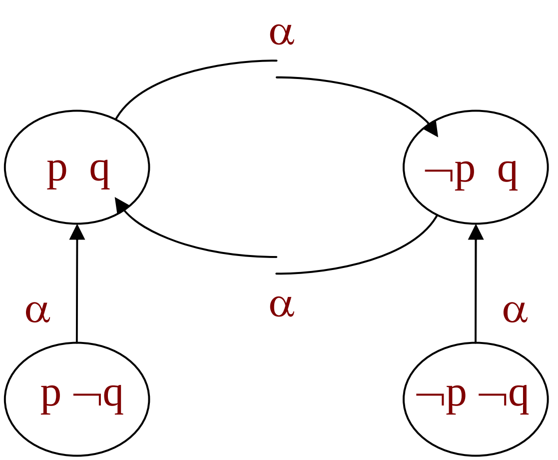
eff (α) = { **if** $p \wedge q$ **then** α **causes** $\neg p$,
if $\neg p \wedge q$ **then** α **causes** p,
(**if** $\top$ **then**) α **causes** q
}

$$\Gamma^{+}(p) = \neg p \wedge q$$

$$\Gamma^{-}(p) = \Gamma^{+}(\neg p) = p \wedge q$$

$$\Gamma^{+}(q) = \top$$

$$\Gamma^{-}(q) = \bot$$



$$\Sigma_{\alpha} : \boxed{q_{t+1} \wedge (p_{t+1} \equiv (p_t \equiv \neg q_t))}$$

Compact expression of ontic action effects

2. causal rules

static laws («instantaneous») causality

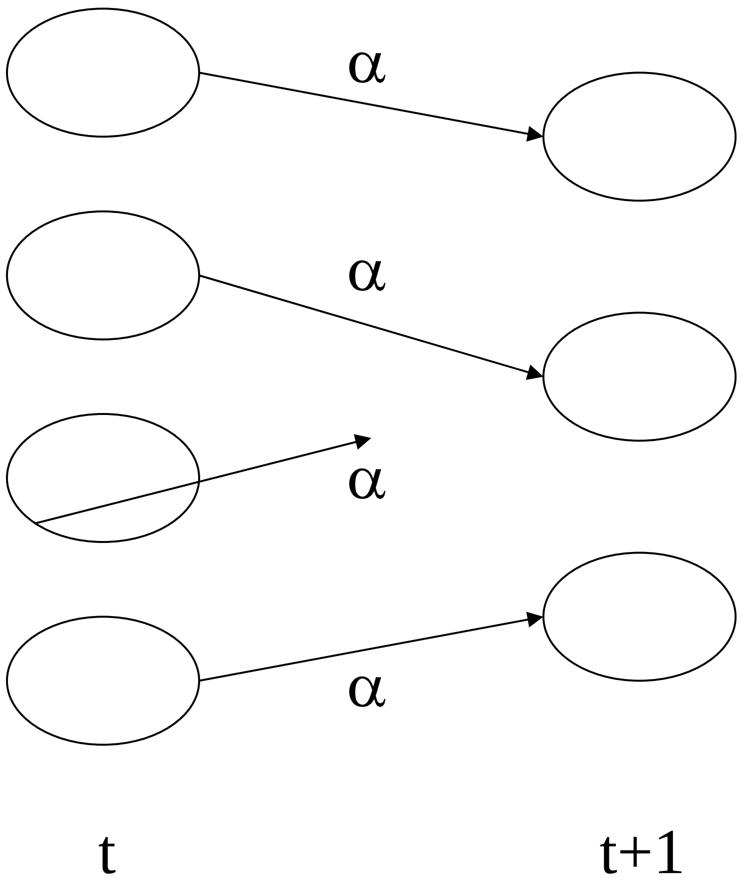
outside $\wedge \neg$ umbrella $\wedge$ rain **causes** $\neg$ dry

+ action laws (dynamic causality)

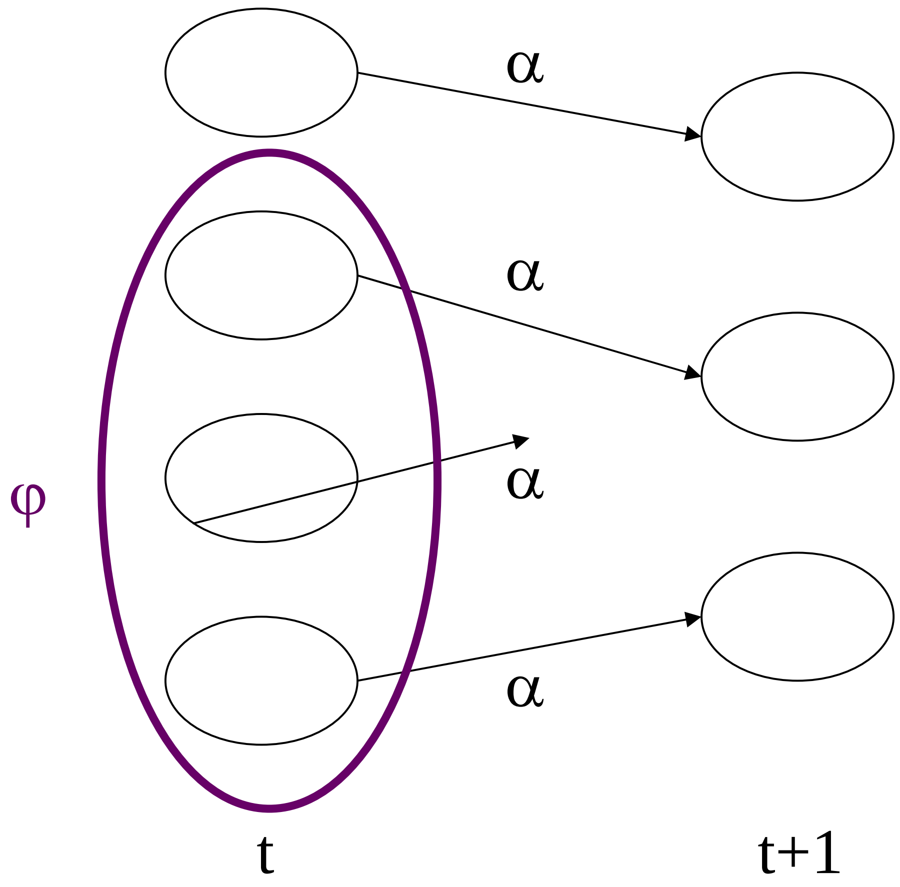
eff (go-out) = {go-out **causes** outside }

$$\Sigma_{\text{go-out}} : \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_{t} \\ \text{rain}_{t+1} \equiv \text{rain}_{t} \\ \text{dry}_{t+1} \equiv \text{dry}_{t} \wedge \neg (\text{outside}_{t+1} \wedge \neg \text{umbrella}_{t+1} \wedge \text{rain}_{t+1}) \end{array}}$$

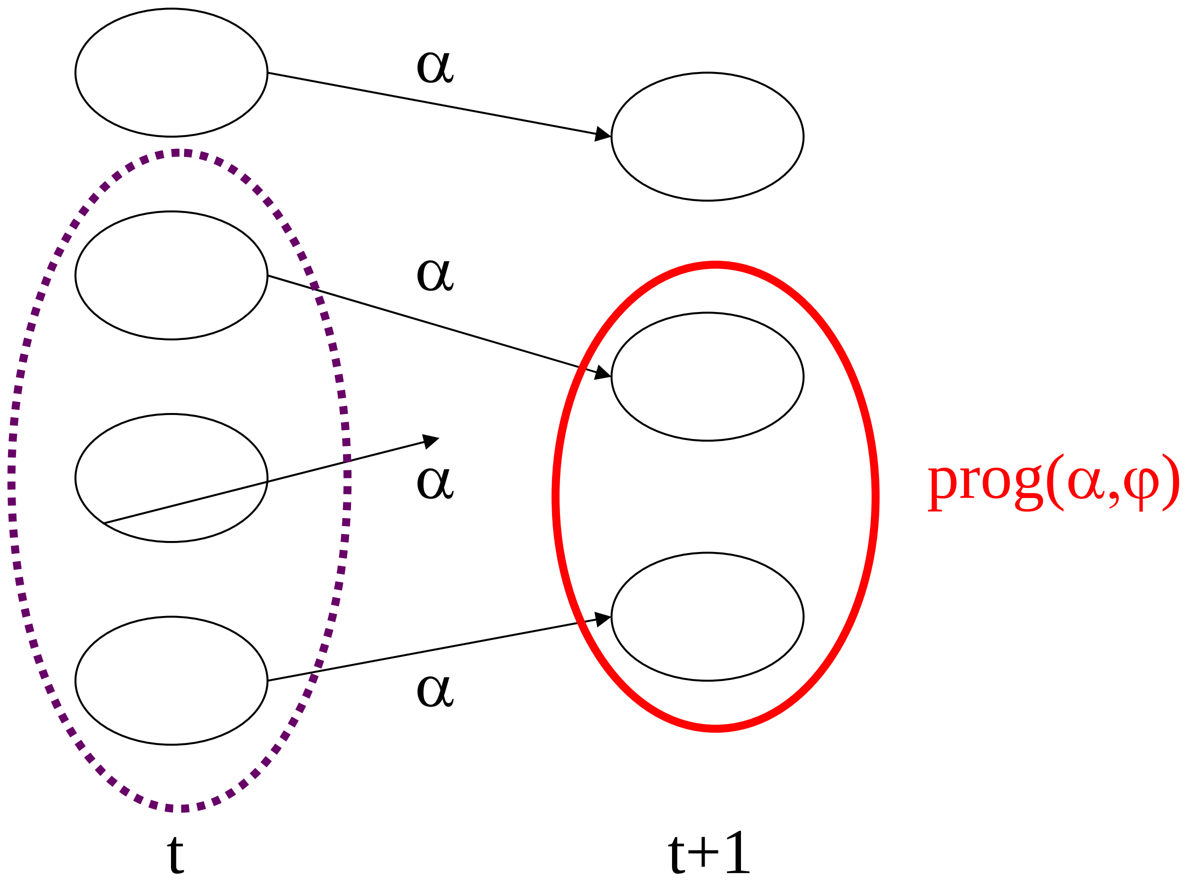
Ontic actions: progression



Ontic actions: progression

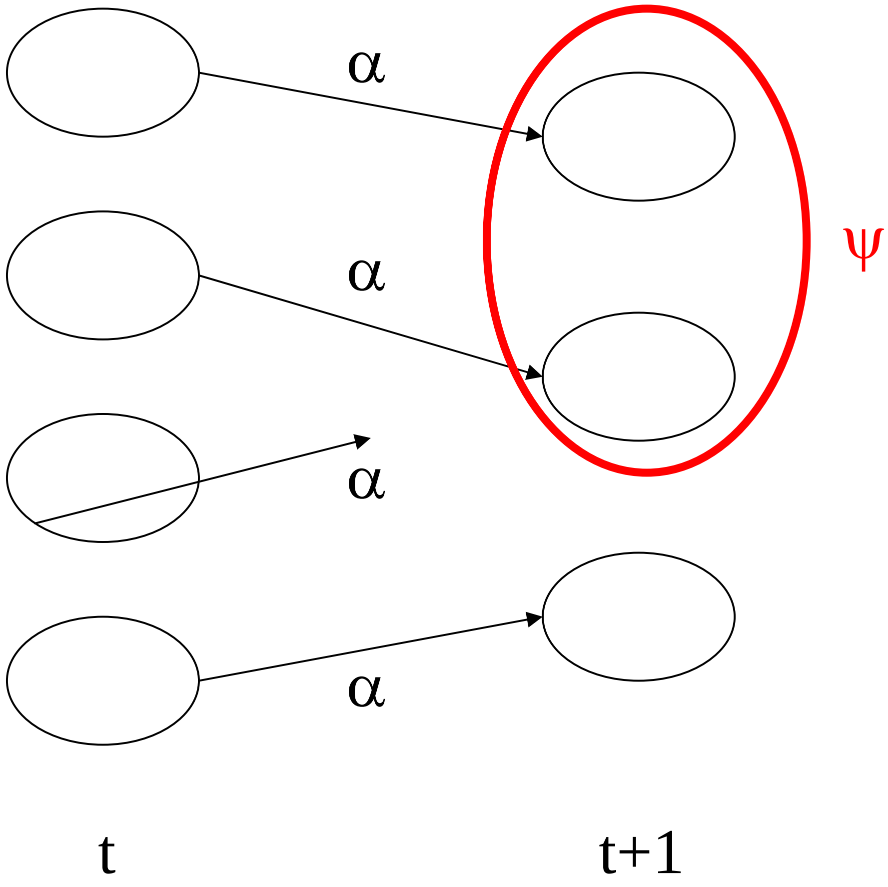


Ontic actions: progression



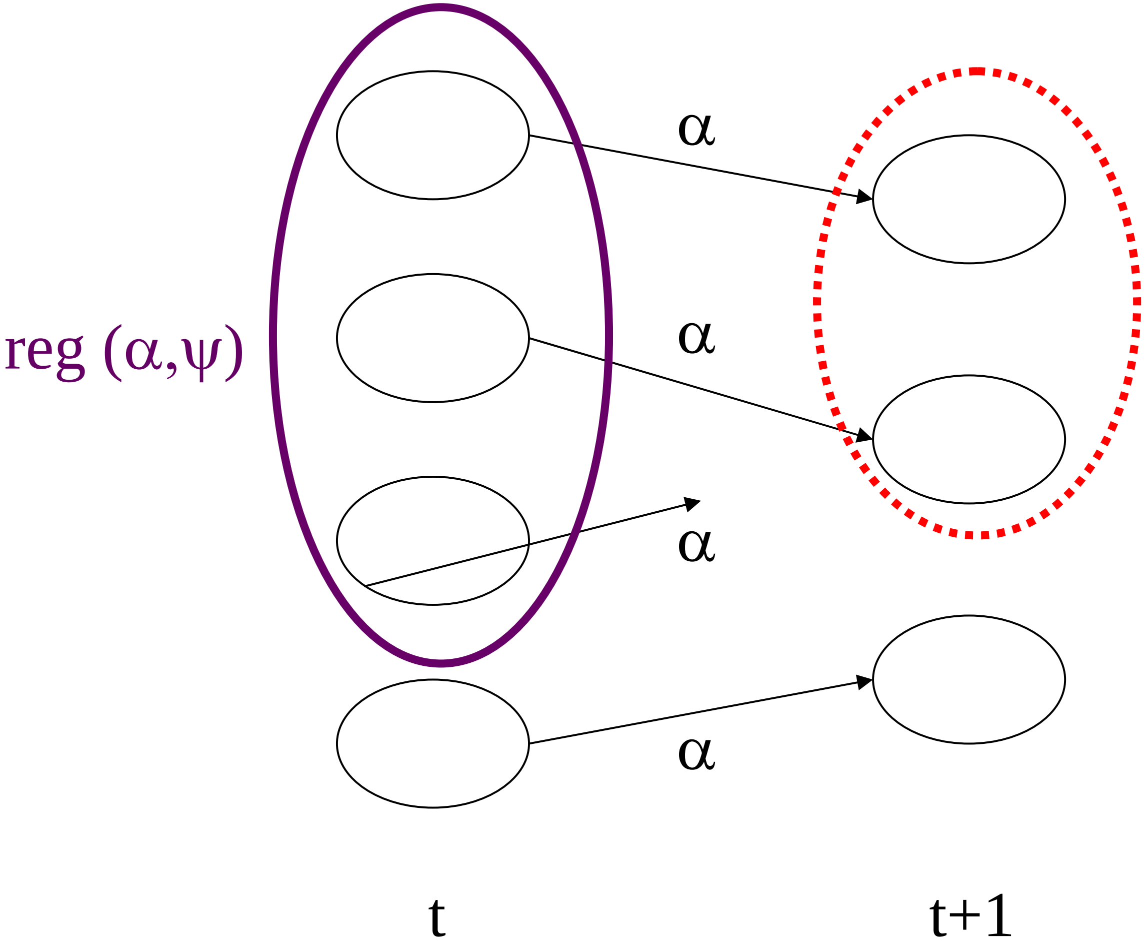
Ontic actions: regression

Regression (*deterministic* actions)



Ontic actions: regression

Regression (*deterministic* actions)



Progression

$\Sigma_{\text{go-out}}$:

$$\boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \land \neg\,(\text{outside}_{t+1} \land \neg\,\text{umbrella}_{t+1} \land \text{rain}_{t+1}\,) \end{array}}$$

$$\varphi_t = \text{dry}_t \land \text{umbrella}_t$$

$$\begin{aligned} \varphi_t \land \Sigma_{\text{go-out}} \equiv\ & \text{outside}_{t+1} \\ & \land \text{umbrella}_t \land \text{umbrella}_{t+1} \\ & \land (\text{rain}_t \equiv \text{rain}_{t+1}\,) \\ & \land \text{dry}_t \land \text{dry}_{t+1} \end{aligned}$$

Progression

$$\Sigma_{\text{go-out}}: \boxed{\begin{array}{l} outside_{t+1} \\ umbrella_{t+1} \equiv umbrella_t \\ rain_{t+1} \equiv rain_t \\ dry_{t+1} \equiv dry_t \land \neg\, (outside_{t+1} \land \neg\, umbrella_{t+1} \land rain_{t+1}\,) \end{array}}$$

$$\varphi_t \;= dry_t \land umbrella_t$$

$$\begin{array}{rl} \varphi_t \land \Sigma_{\text{go-out}} \equiv & outside_{t+1} \\ & \land\, umbrella_t \land umbrella_{t+1} \\ & \land\, (rain_t \equiv rain_{t+1}\,) \\ & \land\, dry_t \land dry_{t+1} \end{array}$$

Progression

$$\Sigma_{go\text{-}out} : \boxed{\begin{array}{l} outside_{t+1} \\ umbrella_{t+1} \equiv umbrella_t \\ rain_{t+1} \equiv rain_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

projection on FL_{t+1}

$$\varphi_t \;= dry_t \wedge umbrella_t$$

$$\begin{aligned} \varphi_t \wedge \Sigma_{go\text{-}out} \equiv \;\; & outside_{t+1} \\ & \wedge umbrella_t \wedge umbrella_{t+1} \\ & \wedge (rain_t \equiv rain_{t+1}\,) \\ & \wedge dry_t \wedge dry_{t+1} \end{aligned}$$

Progression

$\Sigma_{\text{go-out}}$:

$$
\boxed{\begin{array}{l}
outside_{t+1} \\
umbrella_{t+1} \equiv umbrella_t \\
rain_{t+1} \equiv rain_t \\
dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1})
\end{array}}
$$

projection on FL_{t+1}

$\varphi_t = dry_t \wedge umbrella_t$

$$
\varphi_t \wedge \Sigma_{\text{go-out}} \equiv \begin{array}{l}
outside_{t+1} \\
\wedge\, umbrella_t \wedge umbrella_{t+1} \\
\wedge\, (rain_t \equiv rain_{t+1}) \\
\wedge\, dry_t \wedge dry_{t+1}
\end{array}
$$

$$
\begin{array}{l}
outside_{t+1} \\
\wedge\, umbrella_{t+1} \\
\wedge\, dry_{t+1}
\end{array}
$$

projecting on FL_{t+1} = forgetting the symbols in FL_t

Forget (Ø , Σ) = Σ

Forget ({x} , Σ) = $\Sigma_{x = \top} \vee \Sigma_{x = \bot}$

Forget (X $\cup$ {x} , Σ) = Forget ({x}, Forget (X, Σ))

$$\varphi_t \wedge \Sigma_{go\text{-}out} \equiv \begin{array}{l} outside_{t+1} \\ \wedge\ umbrella_t \wedge umbrella_{t+1} \\ \wedge\ (rain_t \equiv rain_{t+1}) \\ \wedge\ dry_t \wedge dry_{t+1} \end{array} \xrightarrow{\text{forget } outside_t} \begin{array}{l} outside_{t+1} \\ \wedge\ umbrella_t \wedge umbrella_{t+1} \\ \wedge\ (rain_t \equiv rain_{t+1}) \\ \wedge\ dry_t \wedge dry_{t+1} \end{array}$$

projecting on FL_{t+1} = forgetting the symbols in FL_t

Forget (Ø , Σ) = Σ

Forget ({x} , Σ) = $\Sigma_{x=\top} \vee \Sigma_{x=\bot}$

Forget (X ∪ {x} , Σ) = Forget ({x}, Forget (X, Σ))

$outside_{t+1}$
$\wedge\ umbrella_{t+1}$
$\wedge\ (rain_t \equiv rain_{t+1})$
$\wedge\ dry_t \wedge dry_{t+1}$

$outside_{t+1}$
$\wedge\ umbrella_t \wedge umbrella_{t+1}$
$\wedge\ (rain_t \equiv rain_{t+1})$
$\wedge\ dry_t \wedge dry_{t+1}$

forget $umbrella_t$

projecting on FL_{t+1} = forgetting the symbols in FL_t

$$\mathrm{Forget}(\varnothing, \Sigma) = \Sigma$$

$$\mathrm{Forget}(\{x\}, \Sigma) = \Sigma_{x=\top} \vee \Sigma_{x=\perp}$$

$$\mathrm{Forget}(X \cup \{x\}, \Sigma) = \mathrm{Forget}(\{x\}, \mathrm{Forget}(X, \Sigma))$$

$$\begin{array}{l} \quad outside_{t+1} \\ \wedge\ umbrella_{t+1} \\ \wedge\ (rain_t \equiv rain_{t+1}) \\ \wedge\ dry_t \wedge dry_{t+1} \end{array} \xrightarrow{\text{forget } rain_t} \begin{array}{l} \quad outside_{t+1} \\ \wedge\ umbrella_{t+1} \\ \wedge\ \top \\ \wedge\ dry_t \wedge dry_{t+1} \end{array}$$

forget $rain_t$

projecting on FL_{t+1} = forgetting the symbols in FL_t

Forget (Ø , Σ) = Σ

Forget ({x} , Σ) = $\Sigma_{x=\top} \vee \Sigma_{x=\bot}$

Forget (X ∪ {x} , Σ) = Forget ({x}, Forget (X, Σ))

$outside_{t+1}$
$\wedge\ umbrella_{t+1}$
$\wedge\ dry_{t+1}$

$outside_{t+1}$
$\wedge\ umbrella_{t+1}$
$\wedge\ dry_t\ \top \wedge\ dry_{t+1}$

forget dry_t

Progression

$\varphi_t = dry_t \wedge umbrella_t$

projection on FL_{t+1}

$outside_{t+1}$
$\wedge\ umbrella_t \wedge umbrella_{t+1}$
$\wedge\ (rain_t \equiv rain_{t+1})$
$\wedge\ dry_t \wedge dry_{t+1}$

$outside_{t+1}$
$\wedge\ umbrella_{t+1}$
$\wedge\ dry_{t+1}$

Progression

$$\Sigma_{\text{go-out}}: \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg (\text{outside}_{t+1} \wedge \neg \text{umbrella}_{t+1} \wedge \text{rain}_{t+1}) \end{array}}$$

$$\varphi_t = \text{dry}_t \wedge \neg \text{umbrella}_t$$

$$\begin{aligned} \varphi_t \wedge \Sigma_{\text{go-out}} \equiv\ & \text{outside}_{t+1} \\ & \wedge \neg \text{umbrella}_t \wedge \neg \text{umbrella}_{t+1} \\ & \wedge (\text{rain}_t \equiv \text{rain}_{t+1}) \\ & \wedge \text{dry}_t \wedge (\text{dry}_{t+1} \equiv \neg \text{rain}_{t+1}) \end{aligned}$$

Progression

$\Sigma_{\text{go-out}}$:

$$\boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\, (\text{outside}_{t+1} \wedge \neg\, \text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$\varphi_t = \text{dry}_t \wedge \neg\, \text{umbrella}_t$

$$\begin{array}{l} \text{outside}_{t+1} \\ \wedge \neg\, \text{umbrella}_t \wedge \neg\, \text{umbrella}_{t+1} \\ \wedge\, (\text{rain}_t \equiv \text{rain}_{t+1}\,) \\ \wedge\, \text{dry}_t \wedge (\text{dry}_{t+1} \equiv \neg\, \text{rain}_{t+1}\,) \end{array}$$

$\varphi_t \wedge \Sigma_{\text{go-out}}$

Progression

$\Sigma_{\text{go-out}}$:

$$\boxed{\begin{array}{l} outside_{t+1} \\ umbrella_{t+1} \equiv umbrella_t \\ rain_{t+1} \equiv rain_t \\ dry_{t+1} \equiv dry_t \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}) \end{array}}$$

$\varphi_t = dry_t \wedge \neg\, umbrella_t$

$$outside_{t+1} \wedge \neg\, umbrella_t \wedge \neg\, umbrella_{t+1} \wedge (rain_t \equiv rain_{t+1}) \wedge dry_t \wedge (dry_{t+1} \equiv \neg\, rain_{t+1})$$

$\varphi_t \wedge \Sigma_{\text{go-out}}$

projection

$$outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge (dry_{t+1} \equiv \neg\, rain_{t+1})$$

Progression

$$\Sigma_{\text{go-out}} : \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$$\varphi_t = \neg\,\text{umbrella}_t$$

$$\begin{array}{l} \text{outside}_{t+1} \\ \wedge \neg\,\text{umbrella}_t \wedge \neg\,\text{umbrella}_{t+1} \\ \wedge\,(\text{rain}_t \equiv \text{rain}_{t+1}\,) \\ \wedge\,(\text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,\text{rain}_{t+1}\,) \end{array}$$

Progression

$$\Sigma_{\text{go-out}}: \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$$\varphi_t = \neg\,\text{umbrella}_t$$

$$\begin{array}{l} \text{outside}_{t+1} \\ \wedge \neg\,\text{umbrella}_t \wedge \neg\,\text{umbrella}_{t+1} \\ \wedge\,(\text{rain}_t \equiv \text{rain}_{t+1}\,) \\ \wedge\,(\text{dry}_{t+1} \rightarrow \text{dry}_t\,) \\ \wedge\,(\text{dry}_{t+1} \rightarrow \neg\,\text{rain}_{t+1}\,) \\ \wedge\,(\text{dry}_t \wedge \neg\,\text{rain}_{t+1} \rightarrow \text{dry}_{t+1}\,) \end{array}$$

Progression

$\Sigma_{\text{go-out}}$:

$$\boxed{\begin{array}{l} outside_{t+1} \\ umbrella_{t+1} \equiv umbrella_t \\ rain_{t+1} \equiv rain_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$\varphi_t = \neg\, umbrella_t$

projection

$$\begin{array}{l} outside_{t+1} \\ \wedge \neg\, umbrella_t \wedge \neg\, umbrella_{t+1} \\ \wedge\, (rain_t \equiv rain_{t+1}\,) \\ \wedge\, (dry_{t+1} \rightarrow dry_t\,) \\ \wedge\, (dry_{t+1} \rightarrow \neg\, rain_{t+1}\,) \\ \wedge\, (dry_t \wedge \neg\, rain_{t+1} \rightarrow dry_{t+1}\,) \end{array}$$

$$\begin{array}{l} outside_{t+1} \\ \wedge \neg\, umbrella_{t+1} \\ \wedge\, (dry_{t+1} \rightarrow \neg\, rain_{t+1}\,) \end{array}$$

Progression: complexity

prog $(\alpha,\varphi) \models \psi$

iff $\varphi_t \wedge \Sigma_\alpha \models \psi_{t+1}$

PROGRESSION is coNP-complete

Regression (deterministic actions)

$$\Sigma_{\text{go-out}} : \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = \text{dry}_{t+1} \wedge \text{rain}_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{\text{go-out}} \equiv\ & \text{outside}_{t+1} \\ & \wedge \text{umbrella}_t \equiv \text{umbrella}_{t+1} \\ & \wedge \text{rain}_t \wedge \text{rain}_{t+1} \\ & \wedge \text{dry}_{t+1} \wedge \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{aligned}$$

Regression (deterministic actions)

$$\Sigma_{\text{go-out}} : \boxed{\begin{array}{l} outside_{t+1} \\ umbrella_{t+1} \equiv umbrella_{t} \\ rain_{t+1} \equiv rain_{t} \\ dry_{t+1} \equiv dry_{t} \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = dry_{t+1} \wedge rain_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{\text{go-out}} \equiv\ & outside_{t+1} \\ & \wedge umbrella_{t} \equiv umbrella_{t+1} \\ & \wedge rain_{t} \wedge rain_{t+1} \\ & \wedge dry_{t+1} \wedge dry_{t} \wedge umbrella_{t+1} \end{aligned}$$

Regression (deterministic actions)

$$\Sigma_{\text{go-out}}: \boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\, (\text{outside}_{t+1} \wedge \neg\, \text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = \text{dry}_{t+1} \wedge \text{rain}_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{\text{go-out}} \equiv\ & \text{outside}_{t+1} \\ & \wedge \text{umbrella}_t \wedge \text{umbrella}_{t+1} \\ & \wedge \text{rain}_t \wedge \text{rain}_{t+1} \\ & \wedge \text{dry}_{t+1} \wedge \text{dry}_t \end{aligned}$$

Regression (deterministic actions)

$\Sigma_{\text{go-out}}$:

$$\boxed{\begin{array}{l} \text{outside}_{t+1} \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{rain}_{t+1} \equiv \text{rain}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}\,) \end{array}}$$

$\Psi_{t+1} = \text{dry}_{t+1} \wedge \text{rain}_{t+1}$

$$\begin{array}{ll} \Psi_{t+1} \wedge \Sigma_{\text{go-out}} \equiv & \text{outside}_{t+1} \\ & \wedge\ \text{umbrella}_t \wedge \text{umbrella}_{t+1} \\ & \wedge\ \text{rain}_t \wedge \text{rain}_{t+1} \\ & \wedge\ \text{dry}_{t+1} \wedge \text{dry}_t \end{array}$$

projection on FL_t

$$\begin{array}{l} \text{umbrella}_t \\ \wedge\ \text{rain}_t \\ \wedge\ \text{dry}_t \end{array}$$

Compact expression of ontic action effects

3. nondeterminism

two possible ways or representating nondeterminism

if pre$_i$ **then** α **causes** γ_1
or causes γ_2
…
or causes γ_n

disjunction of effects

if pre$_i$ **then** α **causes** $\gamma_1 \vee \ldots \vee \gamma_n$

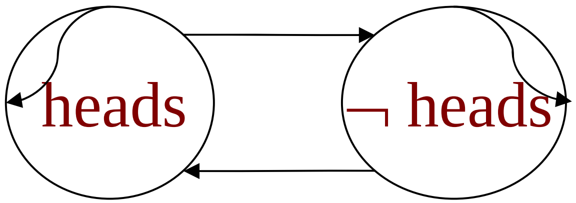
disjunctive effects

Compact expression of ontic action effects

3. non-déterminisme

two possible ways or representating nondeterminism

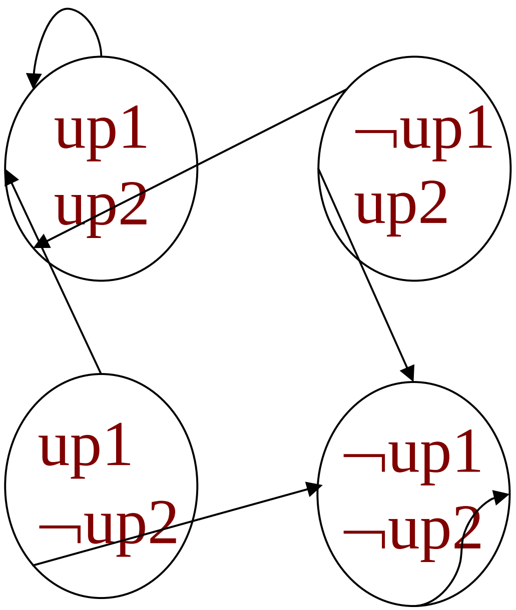
toss-coin **causes** heads
or causes ¬ heads



Compact expression of ontic action effects

3. nondeterminism

two possible ways or representating nondeterminism

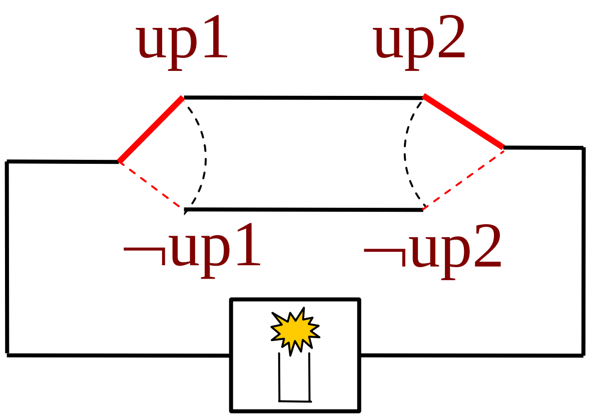


α = make (up1 $\equiv$ up2) true

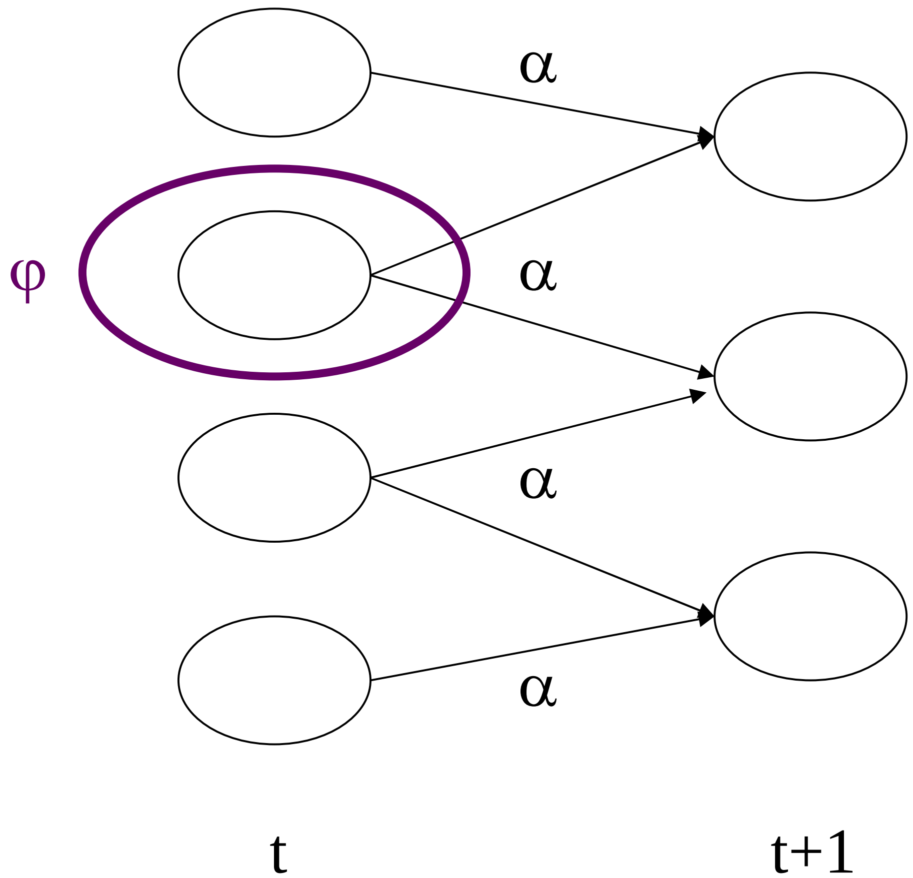
α **causes** up1 $\equiv$ up2

up1 $\equiv$ up2 **causes** light

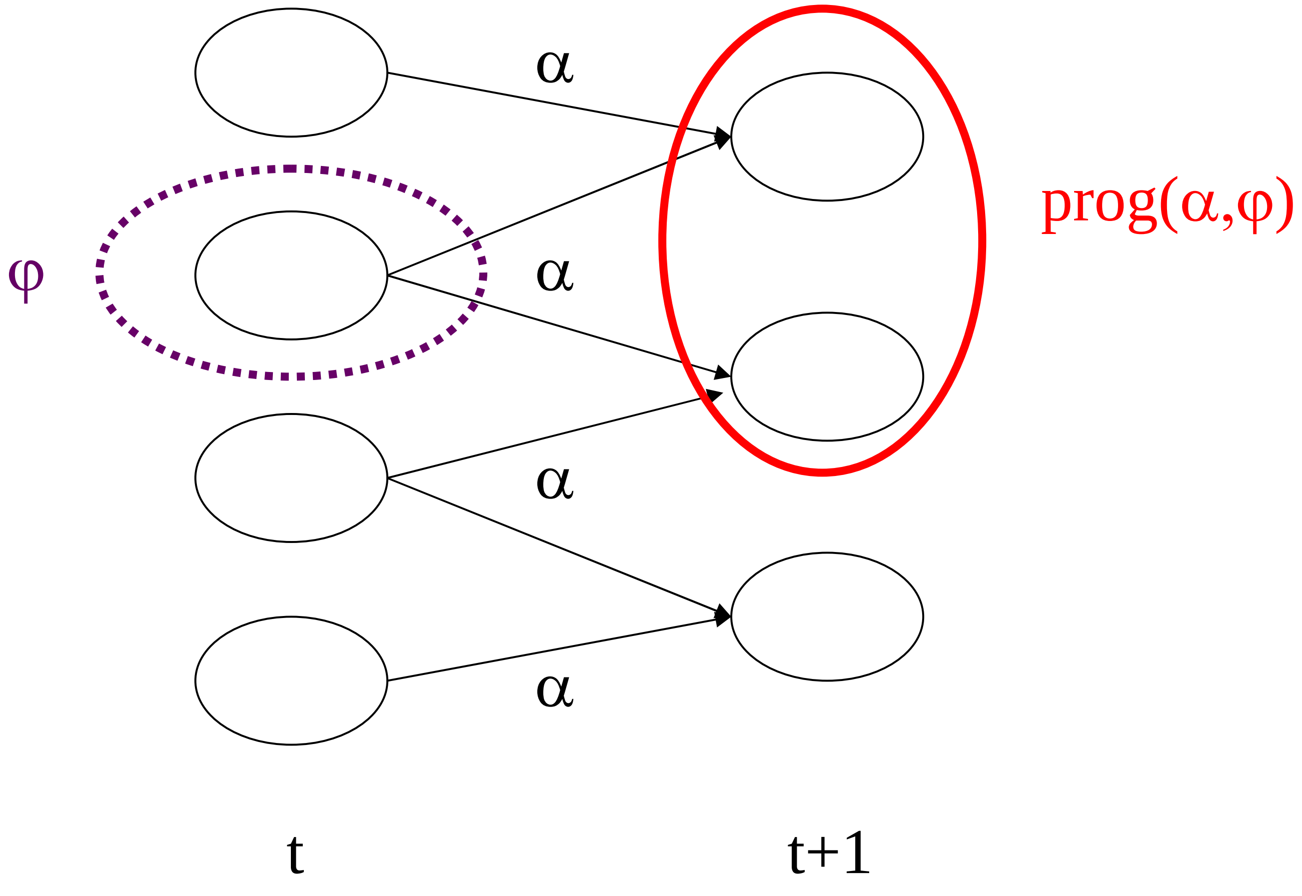
up1 $\equiv$ $\neg$ up2 **causes** $\neg$ light



Progression (nondeterministic actions)



Progression (nondeterministic actions)



Regression (nondeterministic actions) : **two forms**

Weak regression: given

- a belief state ψ_{t+1} at time t+1
- an action α performed between t et t+1

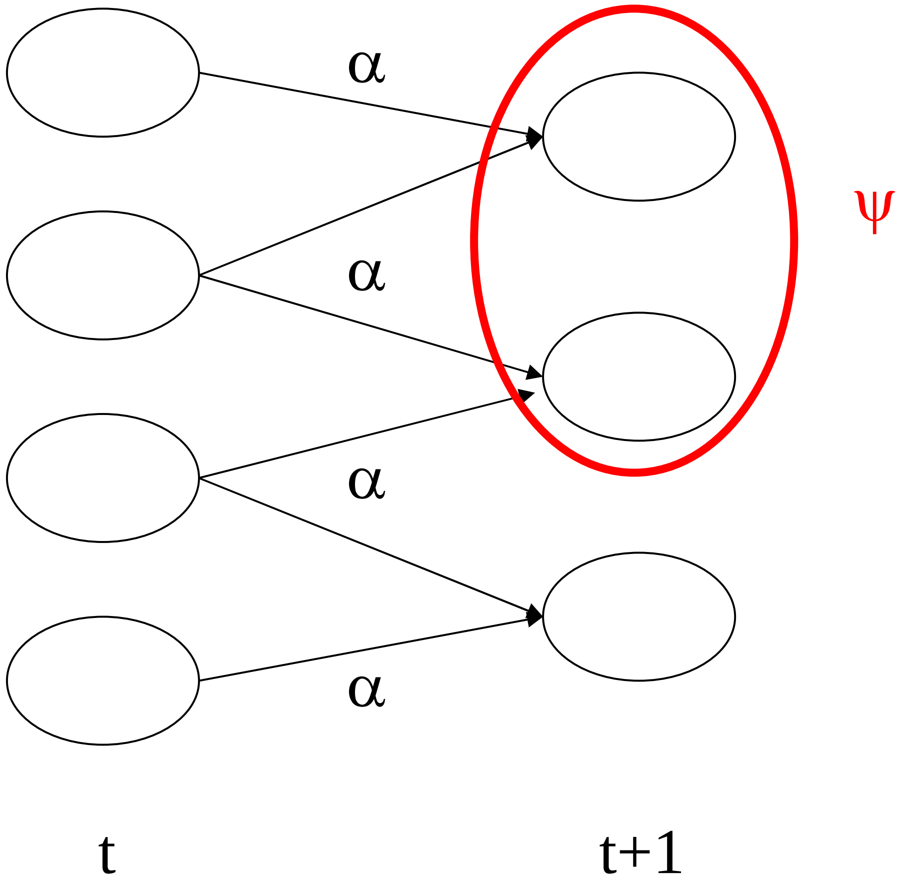
which beliefs can we infer about the state of the world at time t ?

Strong regression: given

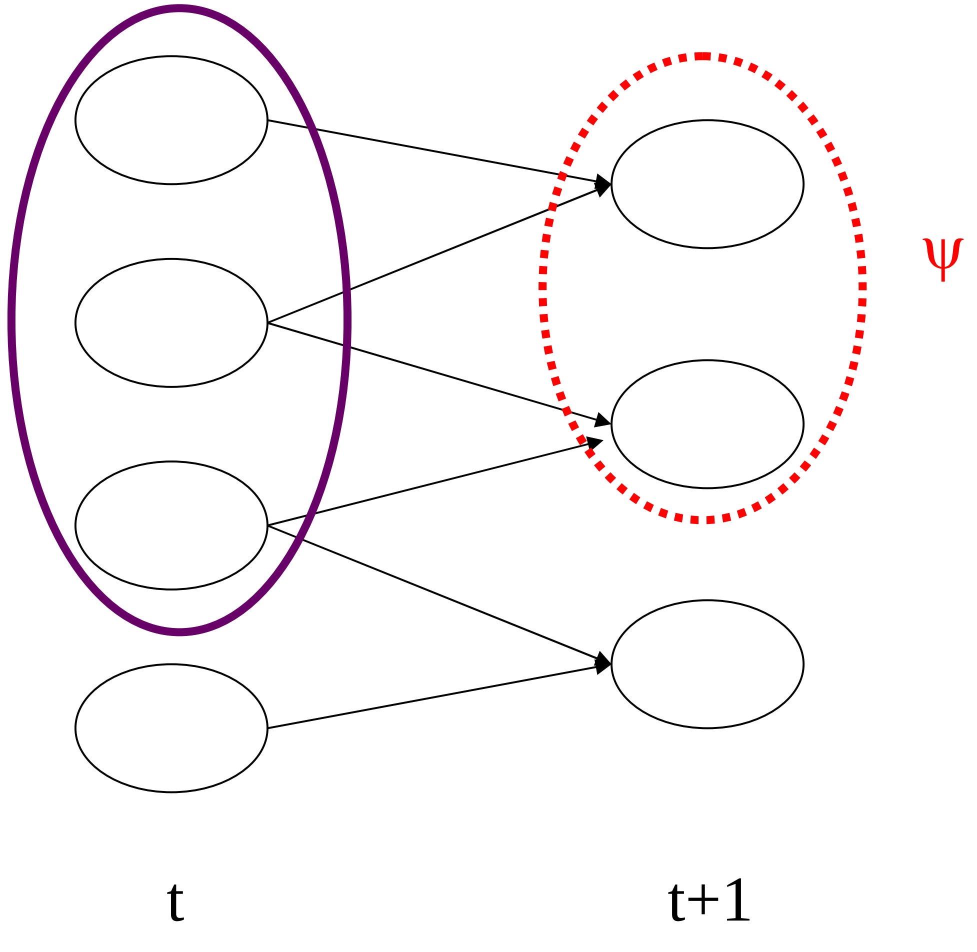
- a goal ψ_{t+1}
- an action α

what are the states of the world in which performing α guarantees to reach ψ_{t+1} ?

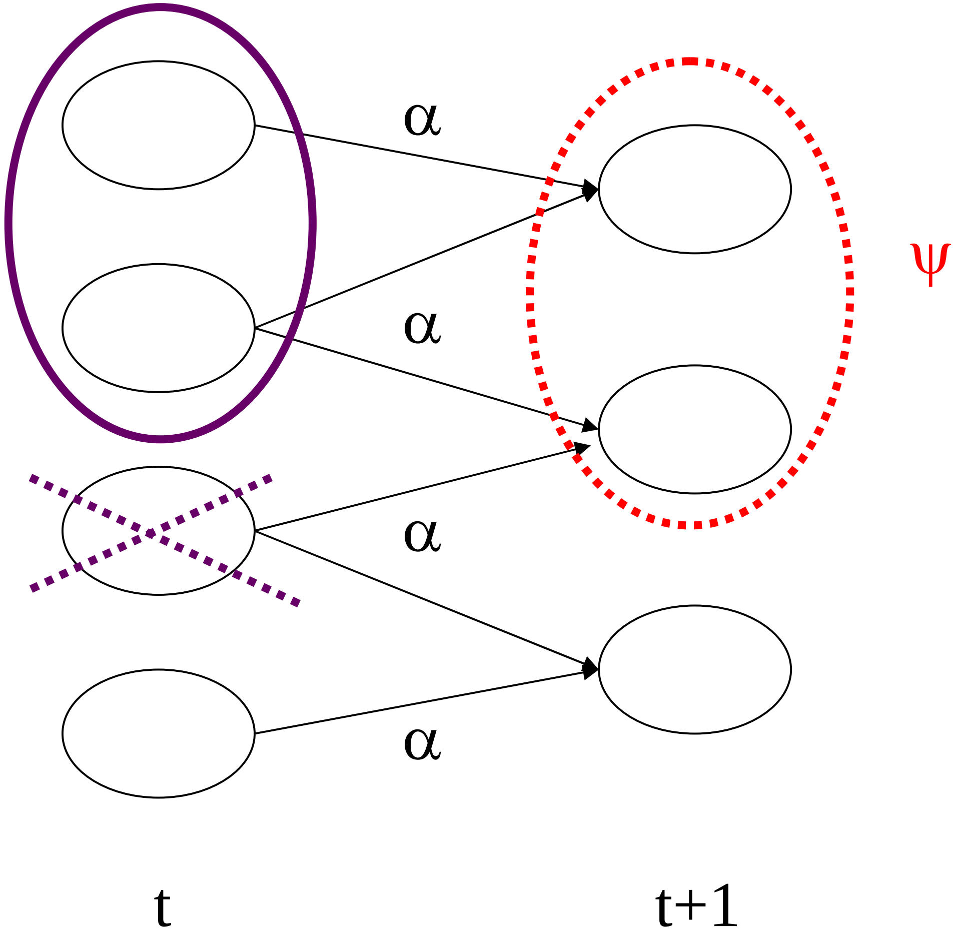
Weak regression (nondeterministic actions)



Weak regression (nondeterministic actions)



Strong regression (nondeterministic actions)



eff (go-out) = {go-out **cause** outside}

eff (wait) = {wait **causes** rain
or causes $\neg$ rain }

outside $\wedge \neg$ umbrella $\wedge$ rain **causes** $\neg$ dry

$$\Sigma_{\text{wait}} : \boxed{\begin{array}{l} \text{outside}_{t+1} \equiv \text{outside}_t \\ \text{umbrella}_{t+1} \equiv \text{umbrella}_t \\ \text{dry}_{t+1} \equiv \text{dry}_t \wedge \neg\,(\text{outside}_{t+1} \wedge \neg\,\text{umbrella}_{t+1} \wedge \text{rain}_{t+1}) \end{array}}$$

Weak regression (nondeterministic actions)

$$\Sigma_{wait}: \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = outside_{t+1} \wedge dry_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{go\text{-}out} \equiv\ & outside_{t+1} \wedge outside_t \\ & \wedge (umbrella_t \equiv umbrella_{t+1}\,) \\ & \wedge dry_{t+1} \wedge dry_t \\ & \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{aligned}$$

Weak regression (nondeterministic actions)

$$\Sigma_{wait}: \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = outside_{t+1} \wedge dry_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{go\text{-}out} \equiv\ & outside_{t+1} \wedge outside_t \\ & \wedge (umbrella_t \equiv umbrella_{t+1}\,) \\ & \wedge dry_{t+1} \wedge dry_t \\ & \wedge (umbrella_{t+1} \vee \neg\, rain_{t+1}\,) \end{aligned}$$

Weak regression (nondeterministic actions)

$$\Sigma_{wait}: \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = outside_{t+1} \wedge dry_{t+1}$$

$$\begin{aligned} \Psi_{t+1} \wedge \Sigma_{go\text{-}out} \equiv\ & outside_{t+1} \wedge outside_t \\ & \wedge (umbrella_t \equiv umbrella_{t+1}\,) \\ & \wedge dry_{t+1} \wedge dry_t \\ & \wedge (umbrella_t \vee \neg\, rain_{t+1}\,) \end{aligned}$$

Weak regression (nondeterministic actions)

Σ_{wait} :

$$\boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$$\Psi_{t+1} = outside_{t+1} \wedge dry_{t+1}$$

$$\begin{array}{rl} \Psi_{t+1} \wedge \Sigma_{attendre} \equiv & outside_{t+1} \wedge outside_t \\ & \wedge\,(umbrella_t \equiv umbrella_{t+1}\,) \\ & \wedge\, dry_{t+1} \wedge dry_t \\ & \wedge\,(umbrella_t \vee \neg\, rain_{t+1}\,) \end{array}$$

projection on FL_t

$dry_t \wedge outside_t$

Strong regression (nondeterministic actions)

Models of StrongReg(a, y) : $\{s \mid prog(\alpha, s) \models \psi\}$

Strong regression (nondeterministic actions)

γ (conjunction of literals) minimal success condition (MSC)

iff $\begin{cases} \text{(1) prog } (\alpha, \gamma) \models \Psi \\ \text{(2) there is no } \gamma' \subset \gamma \text{ such that prog } (\alpha, \gamma) \models \Psi \end{cases}$

iff $\begin{cases} \text{(1) } \gamma_t \wedge \Sigma_\alpha \models \psi_{t+1} \\ \text{(2) there is no } \gamma' \subset \gamma \text{ such that } \gamma_t \wedge \Sigma_\alpha \models \psi_{t+1} \end{cases}$

ABDUCTION

$$\text{StrongReg}(\alpha, \psi) \equiv \vee \{ \gamma_t \mid \gamma_t \text{ MSC for } \alpha, \psi \}$$

Strong regression (nondeterministic actions)

Σ_{wait} :

$$\boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

$\psi_{t+1} = outside_{t+1} \wedge dry_{t+1}$

γ conjunction of literals

γ minimal success condition (CMS)

$$\text{iff} \begin{cases} (1)\ \gamma_t \wedge \Sigma_\alpha \models \psi_{t+1} \\ (2)\ \text{there is no } \gamma' \subset \gamma \text{ such that } \gamma_t \wedge \Sigma_\alpha \models \psi_{t+1} \end{cases}$$

Strong regression (nondeterministic actions)

$\psi_{t+1} = dry_{t+1} \wedge rain_{t+1}$

$$\Sigma_{wait} : \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

Strong regression (nondeterministic actions)

$\psi_{t+1} = dry_{t+1} \wedge rain_{t+1}$

$$\Sigma_{attendre} : \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\, (outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

minimal success conditions : $\{dry_t \wedge outside_t \wedge umbrella_t\,\}$

Strong regression (nondeterministic actions)

$\psi_{t+1} = dry_{t+1} \wedge rain_{t+1}$

Σ_{wait} :

$$\boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

minimal success conditions : $\{dry_t \wedge outside_t \wedge umbrella_t\ \}$

StrongReg $(\psi, wait) = dry \wedge outside \wedge umbrella$

Strong regression (nondeterministic actions)

$\psi_{t+1} = dry_{t+1} \wedge rain_{t+1}$

$$\Sigma_{wait}: \boxed{\begin{array}{l} outside_{t+1} \equiv outside_t \\ umbrella_{t+1} \equiv umbrella_t \\ dry_{t+1} \equiv dry_t \wedge \neg\,(outside_{t+1} \wedge \neg\, umbrella_{t+1} \wedge rain_{t+1}\,) \end{array}}$$

minimal success conditions: $\{dry_t \wedge outside_t \wedge umbrella_t\,\}$

StrongReg (ψ, wait) = dry $\wedge$ outside $\wedge$ umbrella

$\neq$ WeakReg (ψ, wait) = dry $\wedge$ outside

Weak regression: complexity

Weakreg $(\alpha,\varphi) \models \psi$

iff $\psi_{t+1} \wedge \Sigma_\alpha \models \varphi_t$

WEAK REGRESSION is coNP-complete

Strong regression: complexity

StrongReg (α,φ) $\models \psi$

$$\text{StrongReg}(\alpha, \psi) = \{s \mid \text{prog}\,(\alpha, s) \models \psi\}$$

STRONG REGRESSION is Π^2_p-complete

ontic vs. *epistemic* actions

meant to change the state of the world no feedback	meant to bring new information to the agent leave the state of the world unchanged

Any complex action can be decomposed in a (purely) ontic action followed by a (purely) epistemic action

$$\alpha \equiv \alpha_E \circ \alpha_O$$

Belief states

ensemblist

$b \subseteq S$, $b \neq \emptyset$

probabilistic

b proba distribution on S

$$\sum_{s \in S} b(s) = 1$$

ordinal

b preorder on S

$s \geq_b s'$: s more plausible than s'

Belief states

probabilistic

b proba distribution on S

$$\sum_{s \in S} b(s) = 1$$

ensemblist

$b \subseteq S,\ b \neq \varnothing$

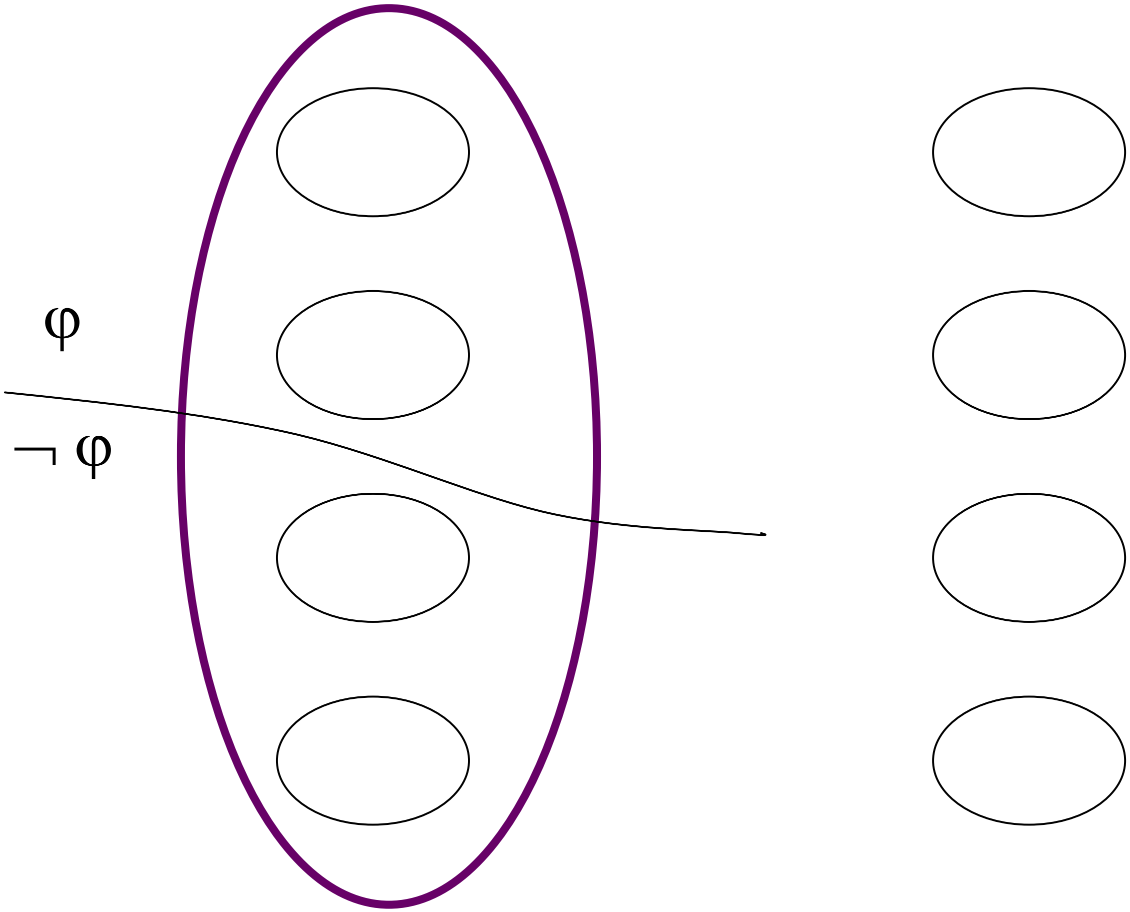
ordinal

b preorder on S

$s \geq_b s'$: s more plausible than s '

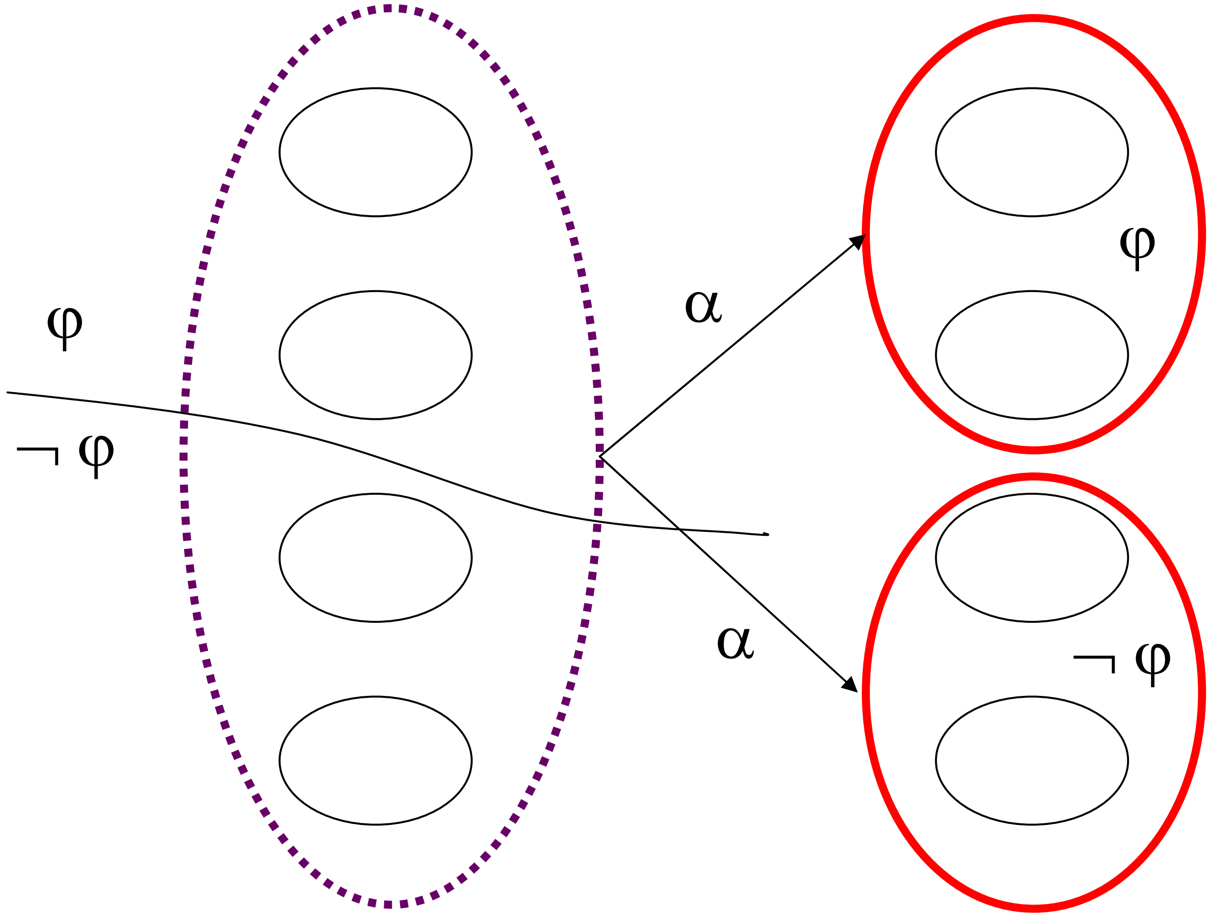
Epistemic actions: progression

α = test (φ)



Epistemic actions: progression

α = test (φ)



Epistemic actions: strong regression

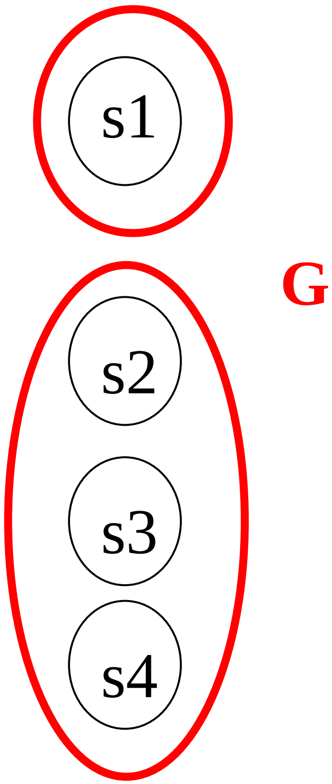
	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

Epistemic actions: strong regression

	φ	$\neg\,\varphi$
ψ	s1	s3
$\neg\,\psi$	s2	s4

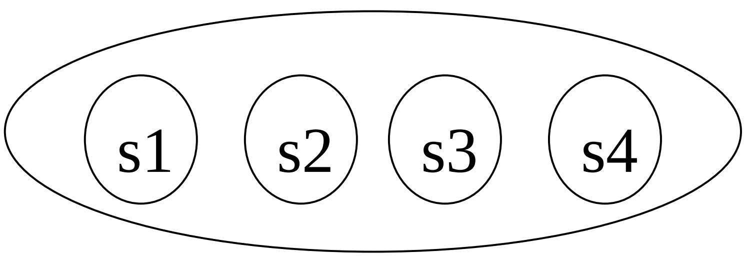
α = test (ψ) = {{s1,s3}, {s2,s4}}

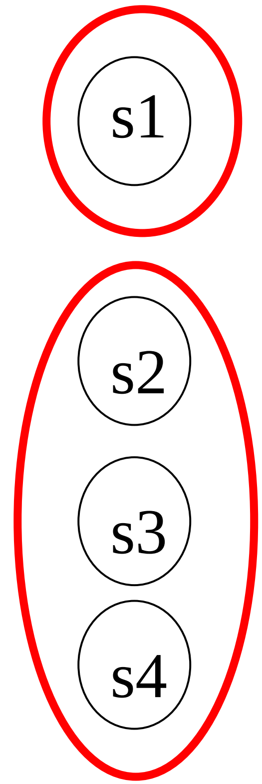


***Epistemic actions:* strong regression**

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

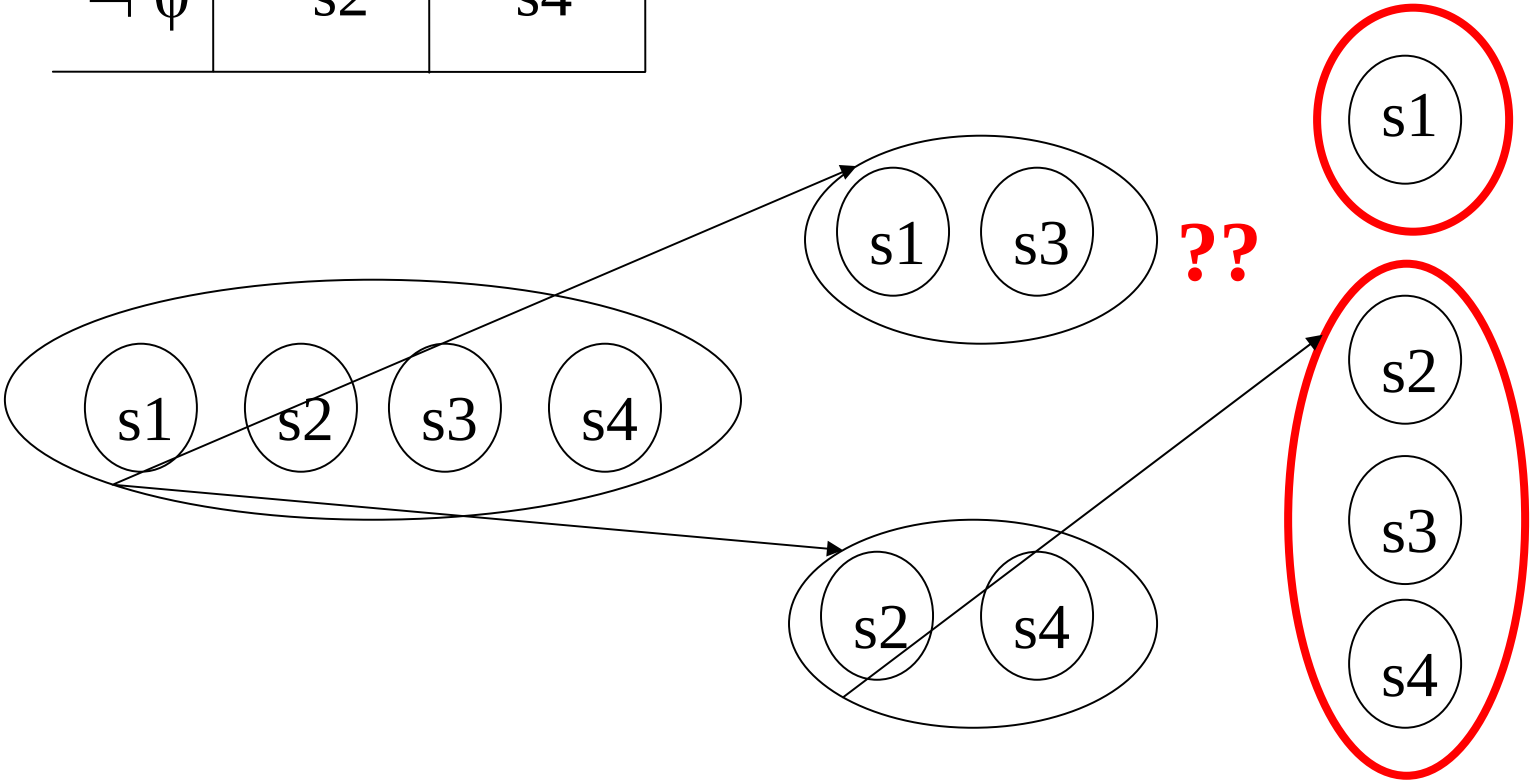




Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

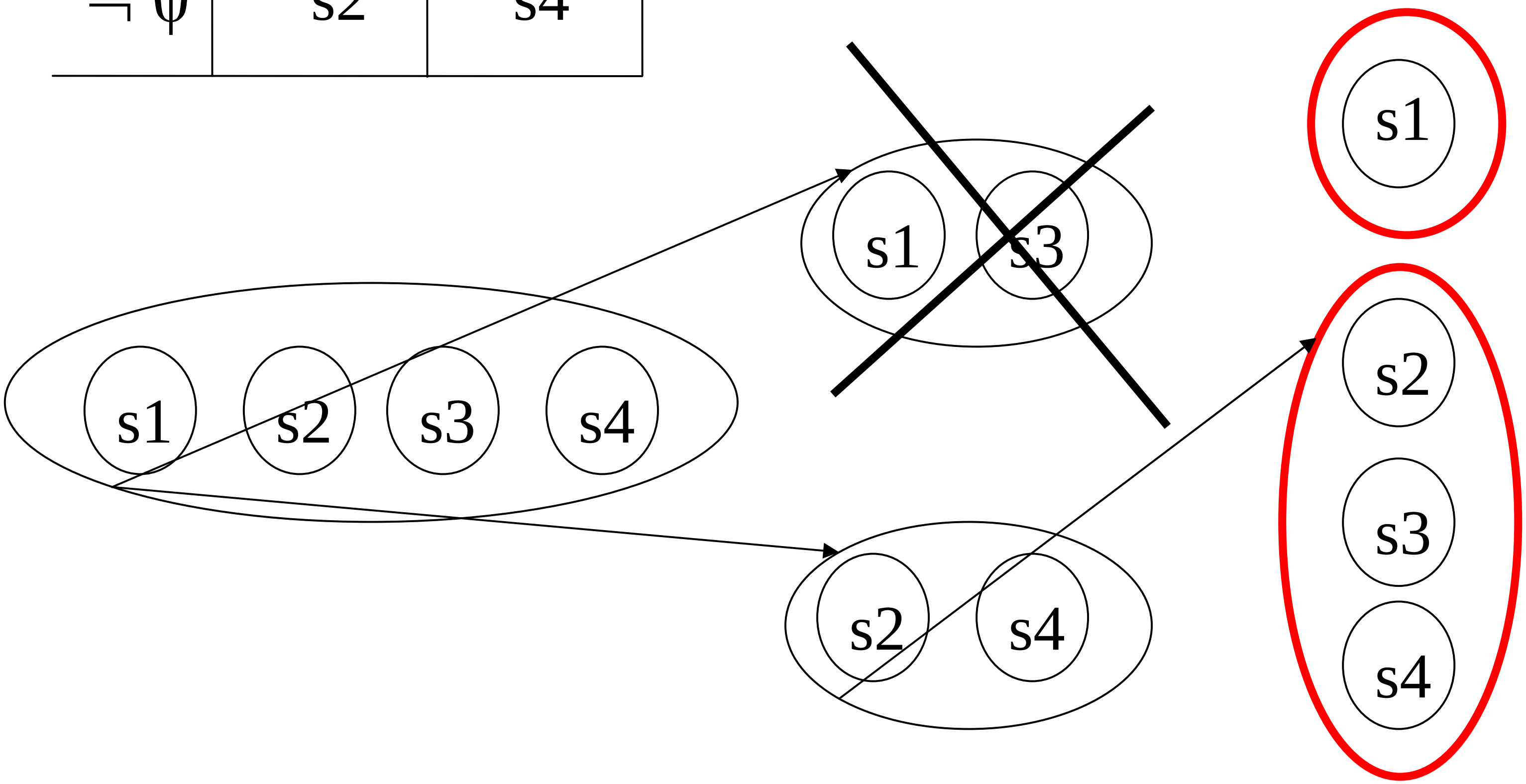
$\alpha = \text{test}(\psi) = \{\{s1,s3\}, \{s2,s4\}\}$



Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

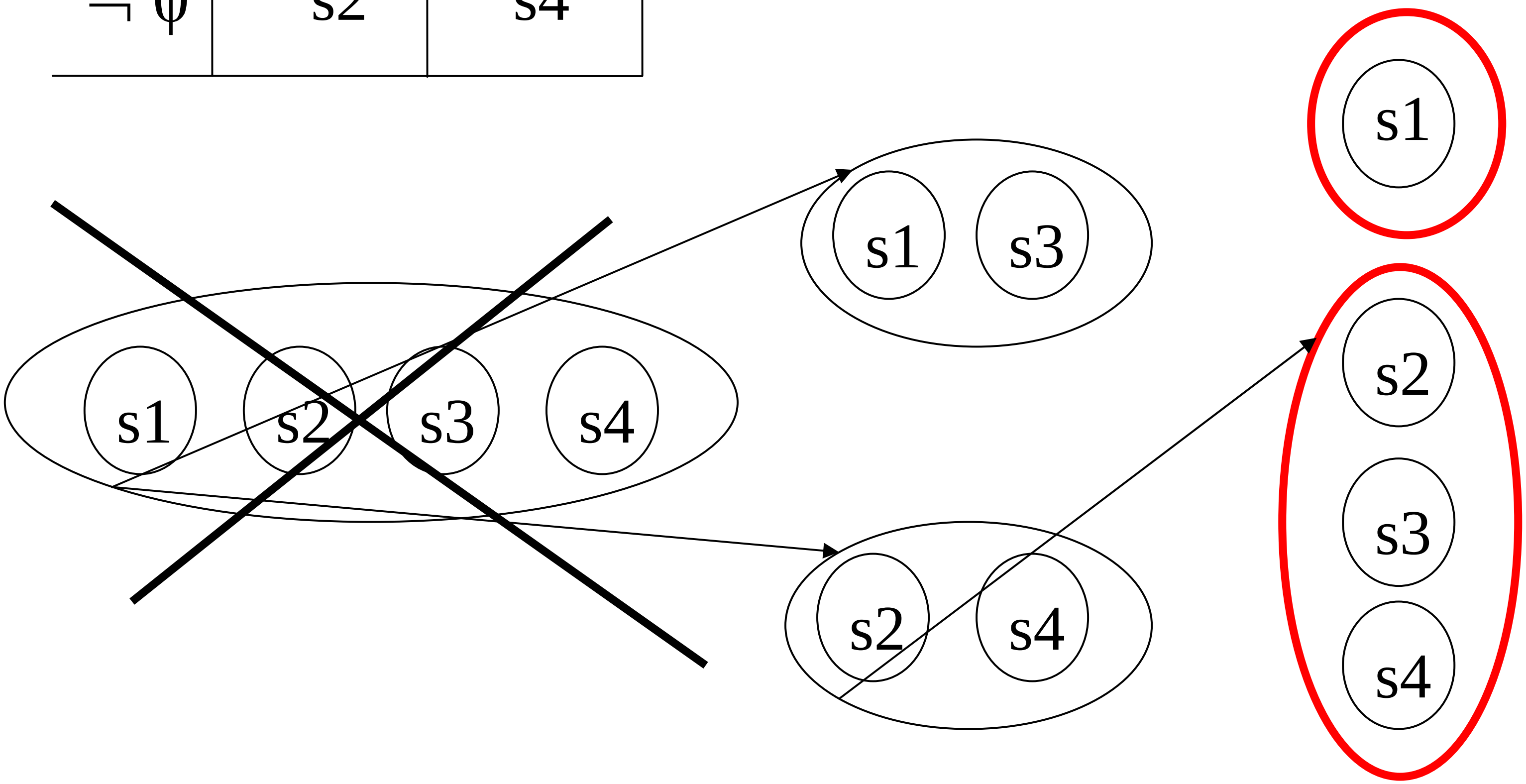
α = test (ψ) = {{s1,s3}, {s2,s4}}



Epistemic actions: **strong regression**

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

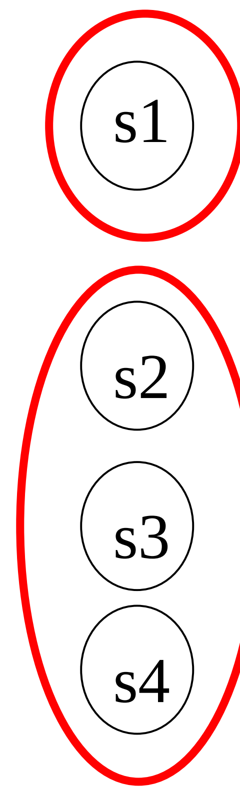
α = test (ψ) = {{s1,s3}, {s2,s4}}

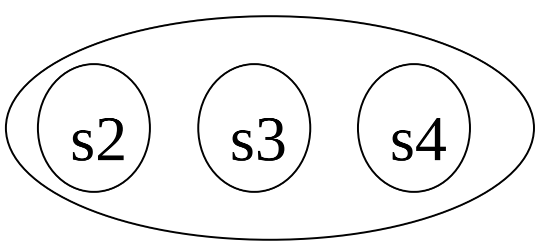


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

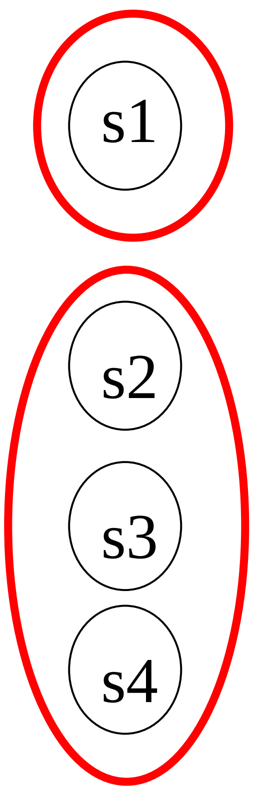


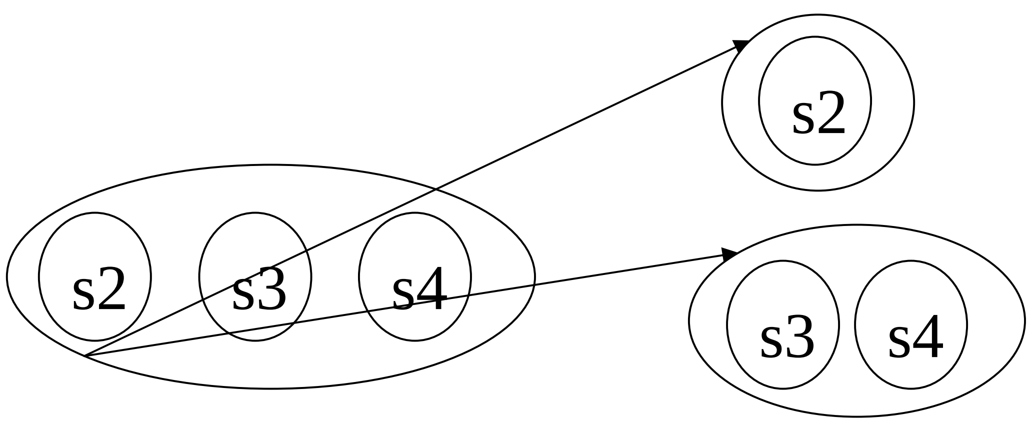


***Epistemic actions:* strong regression**

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

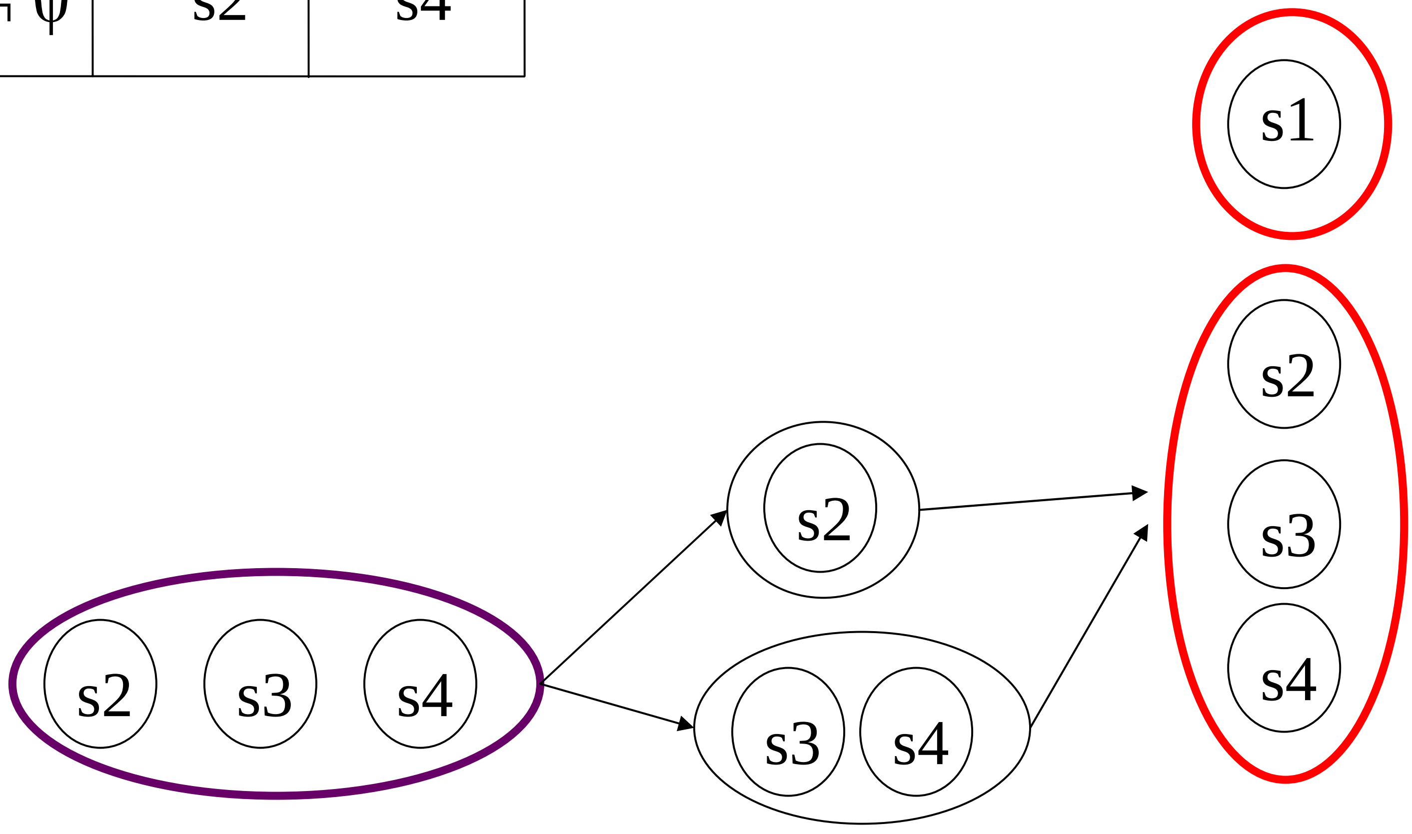




***Epistemic actions:* strong regression**

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

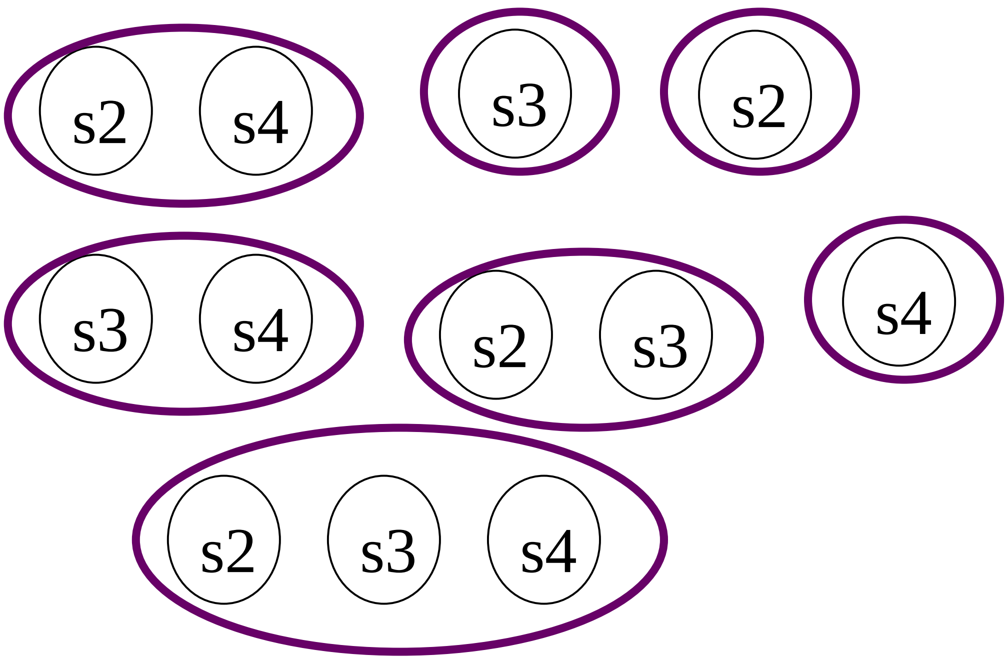
α = test (ψ) = {{s1,s3}, {s2,s4}}

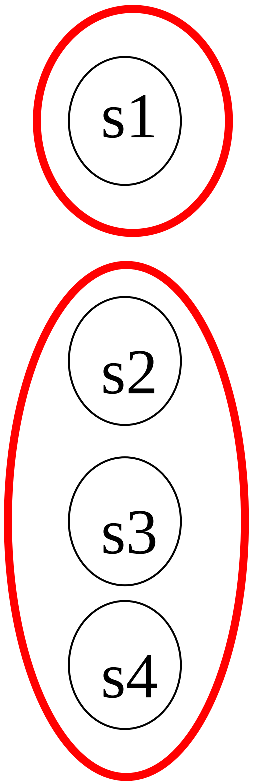


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

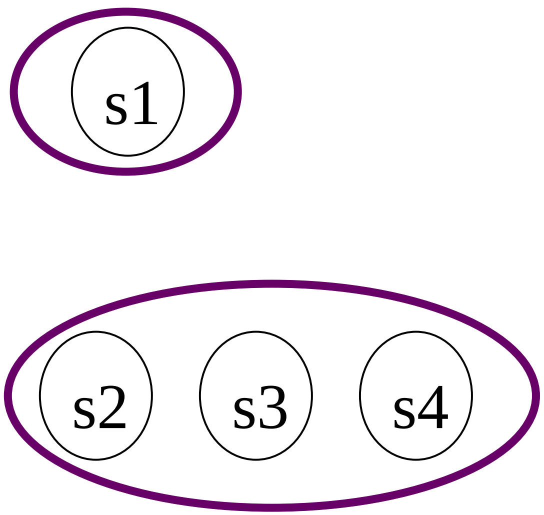


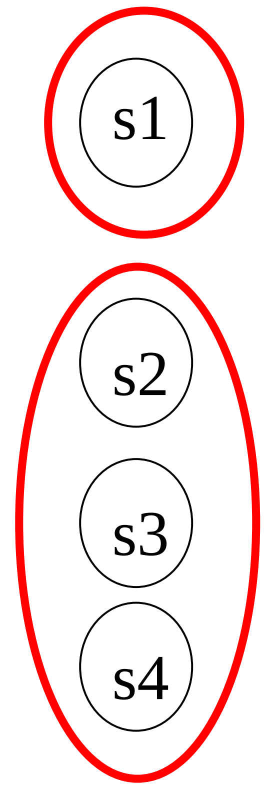


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

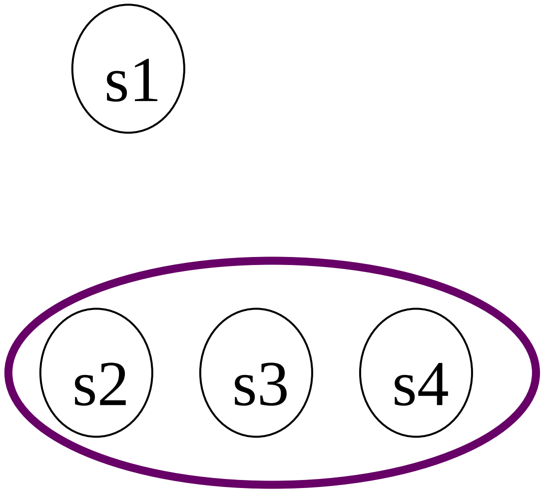


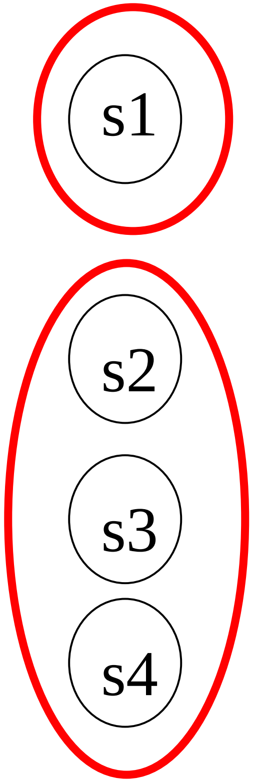


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

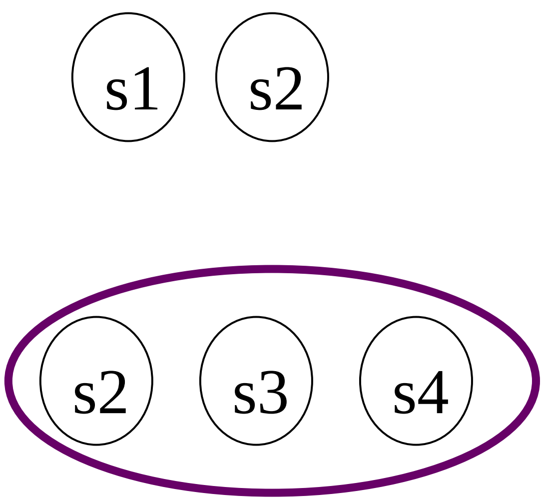


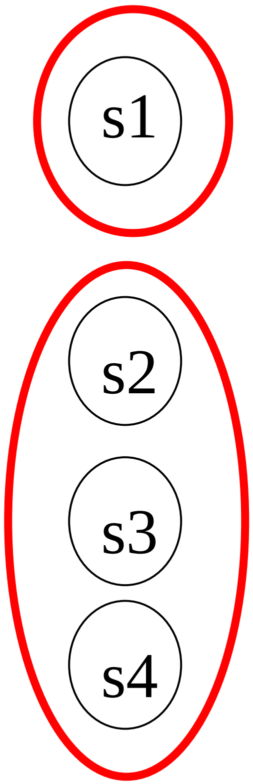


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

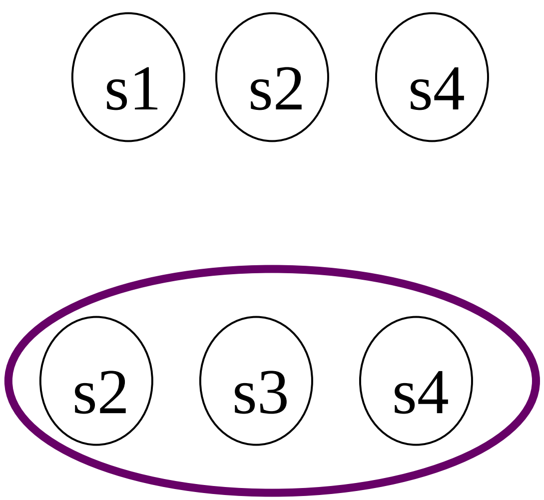


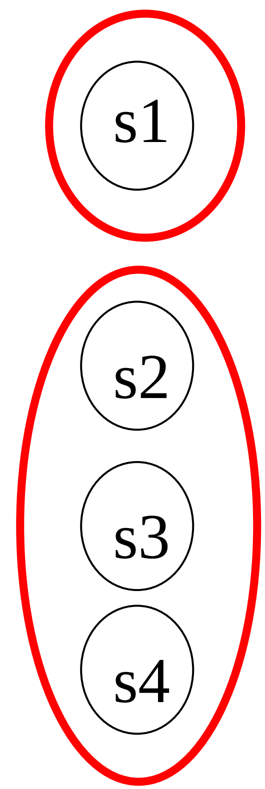


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

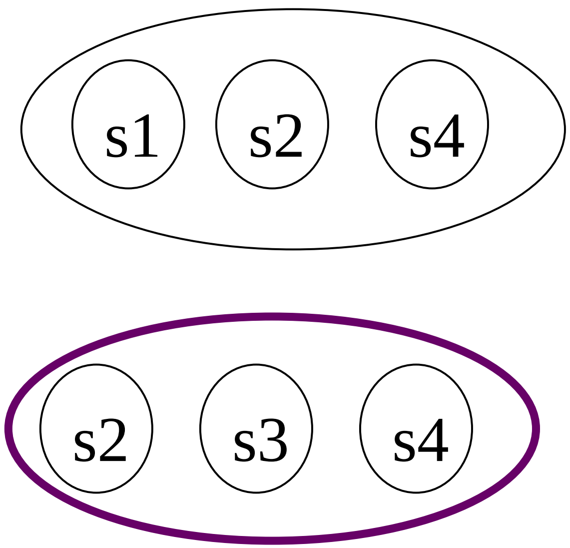


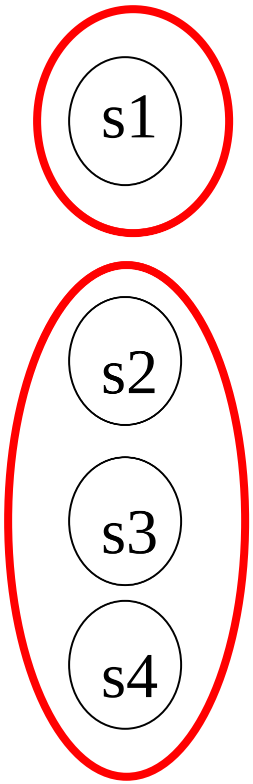


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

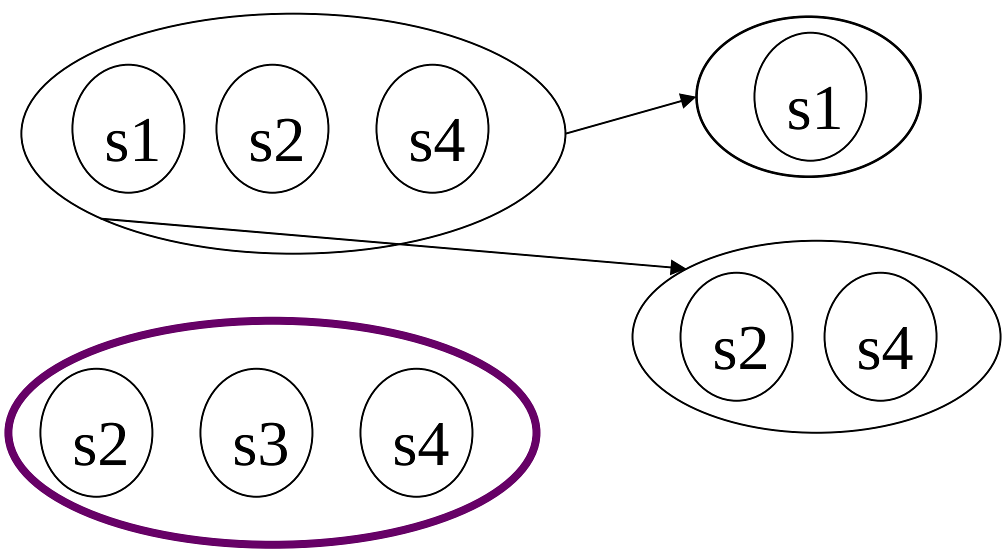


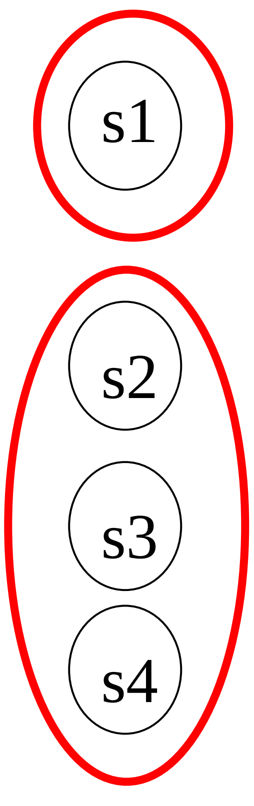


Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

α = test (ψ) = {{s1,s3}, {s2,s4}}

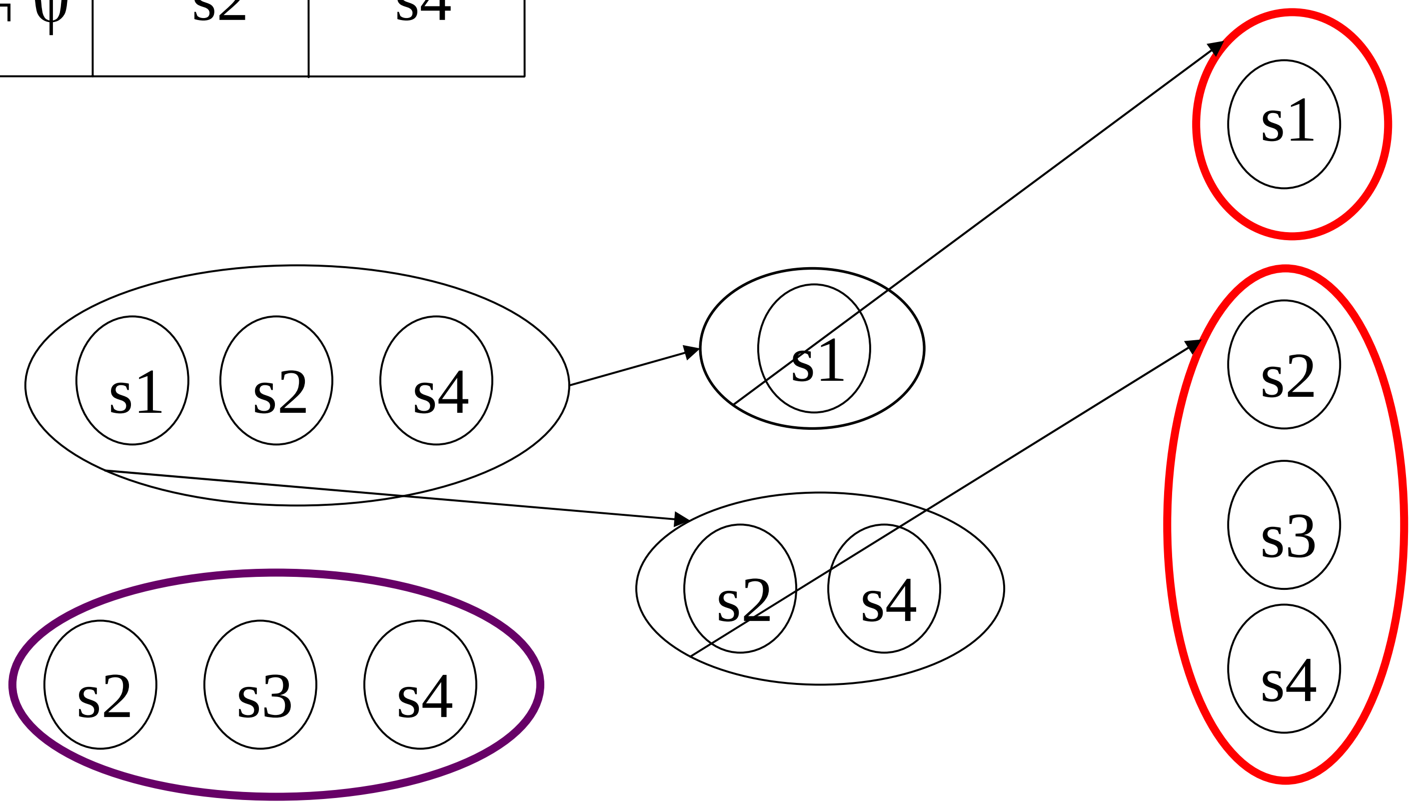




Epistemic actions: strong regression

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

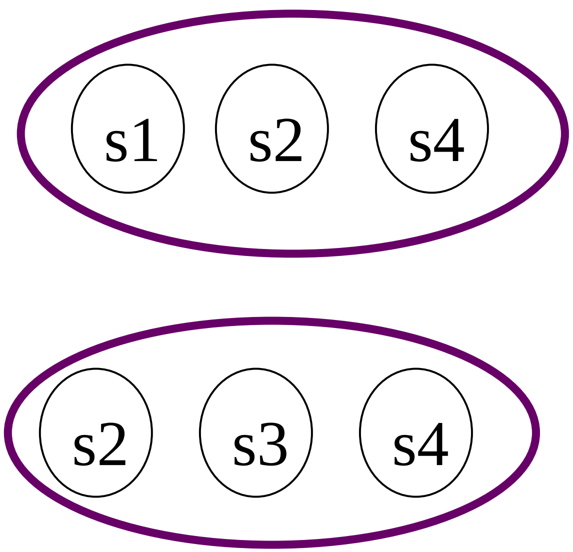
α = test (ψ) = {{s1,s3}, {s2,s4}}

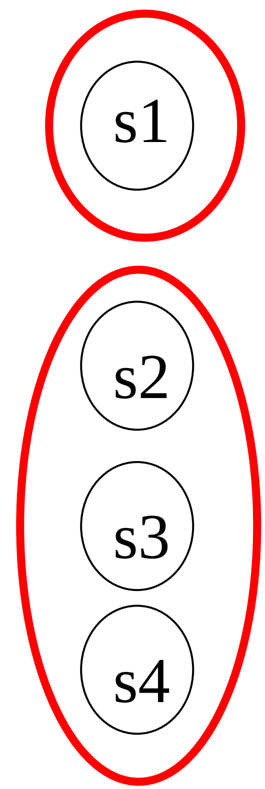


***Epistemic actions:* strong regression**

	φ	$\neg\varphi$
ψ	s1	s3
$\neg\psi$	s2	s4

$$\alpha = \text{test}(\psi) = \{\{s1, s3\}, \{s2, s4\}\}$$





Partial observability and epistemic actions in logic :

making explicit the distinction between facts and knowledge/beliefs

➡ **epistemic/doxastic logics**

Propositional epistemic logic S5

Language $\mathbf{L}_{\mathbf{K,PS}}$:

PS finite set of propositional symbols

- ★ $f \in F$ formula of $\mathbf{L}_{\mathbf{K,PS}}$
- ★ if Φ, Ψ formulas of $\mathbf{L}_{\mathbf{K,PS}}$: $\mathbf{K}\,\Phi$, $\neg\,\Phi$, $\Phi \wedge \Psi$, $\Phi \vee \Psi$ formulas of $\mathbf{L}_{\mathbf{K,PS}}$

$\mathbf{K}\,\Phi$: I know Φ

$\neq$ Φ : Φ is true

Φ *objective* iff Φ is modality-free ➡ Φ denoted by φ

Propositional doxastic logic KD45

Language $\mathbf{L}_{\mathbf{K,PS}}$:

PS finite set of propositional symbols

- $f \in F$ formula of $\mathbf{L}_{\mathbf{K,PS}}$
- if Φ, Ψ formulas of $\mathbf{L}_{\mathbf{K,PS}}$: $\mathbf{B}\,\Phi, \neg\,\Phi, \Phi \wedge \Psi, \Phi \vee \Psi$ formulas of $\mathbf{L}_{\mathbf{K,PS}}$

$\mathbf{B}\,\Phi$: I believe Φ

$\neq \Phi$: Φ is true

Φ *objective* iff Φ is modality-free $\Rightarrow$ Φ denoted by φ

Propositional epistemic logic S5

epistemic atom : $\mathbf{K}\ \varphi$

$$\mathbf{K}\ (\neg\ a \vee b)$$

epistemic formula: Boolean combination of epistemic atoms

$\mathbf{K}\ (\neg\ a \vee b) \vee (\neg\ \mathbf{K}\ c \wedge \neg\ \mathbf{K}\ c)$ but not $c \wedge \mathbf{K}\ (\neg\ a \vee b)$

positive epistemic formula: epistemic formula without any occurrence of $\neg\ \mathbf{K}$

$$\mathbf{K}\ (\neg\ a \vee b) \wedge (\mathbf{K}\ c \vee \mathbf{K}\ \neg\ c)$$

Propositional epistemic logic S5: axiomatics

axiom schemata :

A0. **all tautologies of propositional calculus**

K. $\mathbf{K}\,\Phi \wedge \mathbf{K}\,(\Phi \rightarrow \Psi) \rightarrow \mathbf{K}\,\Psi$ distribution

T. $\mathbf{K}\,\Phi \rightarrow \Phi$ correct beliefs

4. $\mathbf{K}\,\mathbf{K}\,\Phi \leftrightarrow \mathbf{K}\,\Phi$ positive introspection

5. $\mathbf{K}\,\neg\,\mathbf{K}\,\Phi \leftrightarrow \neg\,\mathbf{K}\,\Phi$ negative introspection

inference rules :

MP. from $\vdash \Phi$ and $\vdash \Phi \rightarrow \Psi$ infer $\vdash \Psi$

N. from $\vdash \Phi$ infer $\vdash \mathbf{K}\,\Phi$ necessitation

Propositional doxastic logic KD45: axiomatics

axiom schemata :

A0. **all tautologies of propositional calculus**

K. $\mathbf{K}\,\Phi \wedge \mathbf{K}\,(\Phi \rightarrow \Psi) \rightarrow \mathbf{K}\,\Psi$ distribution

D. $\neg\,\mathbf{K}\,\perp$ consistent beliefs

4. $\mathbf{K}\,\mathbf{K}\,\Phi \leftrightarrow \mathbf{K}\,\Phi$ positive introspection

5. $\mathbf{K}\,\neg\,\mathbf{K}\,\Phi \leftrightarrow \neg\,\mathbf{K}\,\Phi$ negative introspection

inference rules :

MP. from $\vdash \Phi$ and $\vdash \Phi \rightarrow \Psi$ infer $\vdash \Psi$

N. from $\vdash \Phi$ infer $\vdash \mathbf{K}\,\Phi$ necessitation

Propositional S5: semantics

$M = \langle S, val, s^* \rangle$

S nonempty subset of states

$s^* \in S$ (actual state)

$val : S \to (PS \to \{true, false\})$ valuation function

for $p \in PS$: $(M, s) \models p$ iff $val(s)(p) = true$

$(M, s) \models \mathbf{K}\varphi$ iff $(\forall s \in M)(s \models \varphi)$

$(M, s) \models \Phi \wedge \Psi$ iff $(M, s) \models \Phi$ and $(M, s) \models \Psi$

$(M, s) \models \Phi \vee \Psi$ iff $(M, s) \models \Phi$ or $(M, s) \models \Psi$

$(M, s) \models \neg\Phi$ iff $(M, s) \not\models \Phi$

$M \models \Phi$ iff $(M, s^*) \models \Phi$

Propositional KD45: semantics

$M = \langle S, s^*, val \rangle$

S nonempty subset of states;

$s^* \in$ actual state, not necessarily in S

$val : (S \cup \{s^*\}) \rightarrow (PS \rightarrow \{true, false\})$

for $p \in PS$: $(M, s) \models p$ iff $val(s)(p) = true$

$(M, s) \models \mathbf{B}\,\varphi$ iff $(\forall s \in M)(s \models \varphi)$

$(M, s) \models \Phi \wedge \Psi$ iff $(M, s) \models \Phi$ and $(M, s) \models \Psi$

$(M, s) \models \Phi \vee \Psi$ iff $(M, s) \models \Phi$ or $(M, s) \models \Psi$

$(M, s) \models \neg\Phi$ iff $(M, s) \not\models \Phi$

$M \models \Phi$ iff $(M, s^*) \models \Phi$

Propositional epistemic logic S5

Depth of a formula :

$\text{depth}(p) = 0$ for $p \in PS$

$\text{depth}(\neg \Phi) = \text{depth}(\neg \Phi)$

$\text{depth}(\Phi \vee \Psi) = \max(\text{depth}(\Phi), \text{depth}(\Psi))$

$\text{depth}(\mathbf{K}\,\Phi) = 1 + \text{depth}(\Phi)$

A fundamental property :

for all $\Phi \in \mathbf{L}_{\mathbf{K,PS}}$ there exists a $\Phi' \in \mathbf{L}_{\mathbf{K,PS}}$

such that $\models_{S5} \Phi \leftrightarrow \Phi'$ and $\text{depth}(\Phi') \leq 1$

without loss of generality, only objective formulas in the scope of **K**

$\mathbf{K}\ \Phi$

$\mathbf{K}\ \varphi$

Similar result holds in KD45

Propositional epistemic logic S5

Important tautologies and non-tautologies

$$\models_{S5} \mathbf{K}(\varphi \wedge \psi) \leftrightarrow \mathbf{K}\varphi \wedge \mathbf{K}\psi$$

$$\cancel{\models_{S5} \mathbf{K}(\varphi \vee \psi) \leftrightarrow \mathbf{K}\varphi \vee \mathbf{K}\psi}$$

$$\models_{S5} \neg(\mathbf{K}\varphi \wedge \mathbf{K}\neg\varphi)$$

Propositional S5 : decidability

Proposition 1:

there is an algorithm that,
given a model M, a state $s \in M$ and a formula $\Phi \in \mathbf{L}_{\mathbf{K,PS}}$,
determines in time $O(|M|.|\Phi|)$ whether $(M,s) \models_{S5} \Phi$

Proposition 2:

if $\Phi \in \mathbf{L}_{\mathbf{K,PS}}$ is satisfiable in S5
then Φ is satisfiable in a model with at most $|\Phi|$ states

Corollary:

satisfiability in S5 is decidable

Similar result holds in KD45

Propositional S5 : complexity

Proposition 2:

if $\Phi \in \mathbf{L}_{\mathbf{K,PS}}$ is satisfiable in S5

then Φ is satisfiable in a model with at most $|\Phi|$ states

Corollary:

satisfiability in S5 is in **NP**

satisfiability in S5 is **NP-complete**

Similar result holds in KD45

S5 and reasoning about action / planning

on-line reasoning (plan execution)

$\mathbf{K}\top$

$\alpha = \text{test}(\psi)$

simple knowledge states

(epistemic atoms)

$\mathbf{K}\,\psi$ $\qquad$ $\mathbf{K}\,\neg\psi$

off-line reasoning (plan generation)

$\mathbf{K}\top$

$\alpha = \text{test}(\psi)$

$\mathbf{K}\,\psi \vee \mathbf{K}\,\neg\psi$

complex knowledge state

Simple knowledge state (SKS) = epistemic atom $\mathbf{K}\ \varphi$

Complex knowledge state (CKS) = positive epistemic formula

Any CKS can be equivalently written as a disjunction of SKS

$$\Phi \equiv \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$$

$$\mathbf{K}\ a \wedge (\mathbf{K}\ b \vee \mathbf{K}\ \neg b) \equiv \mathbf{K}\ (a \wedge b) \vee \mathbf{K}\ (a \wedge \neg b)$$

Φ CKS

α action (ontic or epistemic)

Progression of a CKS by an action

Prog (Φ, α) strongest formula known to hold
after α is performed,
given that Φ holds initially

Goal regression of a CKS by an action

Reg (Φ, α) weakest formula
whose progression by α entails Φ

Ontic actions in S5 : progression

$\Phi = \mathbf{K}\,\varphi_1 \vee \ldots \vee \mathbf{K}\,\varphi_n$ CKS $\qquad$ α ontic action

$$\boxed{\mathrm{Prog}_O(\Phi, \alpha) = \mathbf{K}\,\mathrm{prog}(\varphi_1, \alpha) \vee \ldots \vee \mathbf{K}\,\mathrm{prog}(\varphi_n, \alpha)}$$

classical progression

$\mathrm{Prog}_O(\Phi, \alpha)$ is a CKS

$$\mathrm{Prog}_O : \mathrm{CKS} \times A_O \to \mathrm{CKS}$$

Ontic actions in S5: regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$\boxed{\mathrm{Reg}_O\ (\Phi, \alpha) = \mathbf{K}\ \mathrm{reg}\ (\varphi_1, \alpha) \vee \ldots \vee \mathbf{K}\ \mathrm{reg}\ (\varphi_n, \alpha)}$$

$\mathrm{Reg}_O : \mathrm{CKS} \times A_O \to \mathrm{CKS}$

Computing regression:

compute *minimal success conditions*

➡ abduction

Epistemic actions in S5 : progression

Epistemic action :

β ➡ $o_1, \ldots, o_p$ possible observations

such that $(o_1 \vee \ldots \vee o_p)$ tautology

$$\boxed{Prog_E(\Phi, \beta) = \Phi \wedge (\mathbf{K}\, o_1 \vee \ldots \vee \mathbf{K}\, o_n)}$$

$$Prog_E : CKS \times A_E \rightarrow CKS$$

Binary tests: $\beta = test(\varphi)$ returns the truth value of φ

$$Prog_E(\Phi, test(\varphi)) = \Phi \wedge (\mathbf{K}\, \varphi \vee \mathbf{K}\, \neg \varphi)$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\, \varphi_1 \vee \ldots \vee \mathbf{K}\, \varphi_n$ complex knowledge state

$\alpha = \text{test}\ (\psi)$

$$\boxed{Reg_E\ (\Phi, \alpha) = \vee \ \{\ \mathbf{K}\ ((\psi \rightarrow \varphi_i) \wedge (\neg \psi \rightarrow \varphi_j)) \mid i,j \in 1 \,..\, n\}}$$

$Reg_E : CKS \times ACT_O \rightarrow CKS$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$\mathrm{Reg}_E\,(\Phi, \alpha) = \bigvee \left\{ \mathbf{K}\,((\psi \rightarrow \varphi_i) \wedge (\neg\,\psi \rightarrow \varphi_j)) \,\middle|\, i,j \in 1 \,..\, n \right\}$$

$\Phi = \mathbf{K}\ v \vee \mathbf{K}\ \neg\, v$ α = test (ψ) = test ($u \wedge v$)

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\,\varphi_1 \vee \ldots \vee \mathbf{K}\,\varphi_n$ complex knowledge state

$$\mathrm{Reg}_E(\Phi, \alpha) = \bigvee \left\{ \mathbf{K}\,((\psi \to \varphi_i) \wedge (\neg\,\psi \to \varphi_j)) \,\middle|\, i,j \in 1 \,..\, n \right\}$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}\, \neg\, v$ $\alpha = \text{test}\,(\psi) = \text{test}\,(u \wedge v)$

$$\begin{aligned}
\mathrm{Reg}_E(\Phi, \alpha) = \; & \mathbf{K}\,((u \wedge v \to v) \wedge ((\neg u \vee \neg v) \to v))\\
\vee\, & \mathbf{K}\,((u \wedge v \to v) \wedge ((\neg u \vee \neg v) \to \neg v))\\
\vee\, & \mathbf{K}\,((u \wedge v \to \neg v) \wedge ((\neg u \vee \neg v) \to v))\\
\vee\, & \mathbf{K}\,((u \wedge v \to \neg v) \wedge ((\neg u \vee \neg v) \to \neg v))
\end{aligned}$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\,\varphi_1 \vee \ldots \vee \mathbf{K}\,\varphi_n$ complex knowledge state

$$\mathrm{Reg}_E(\Phi, \alpha) = \bigvee \{\mathbf{K}\,((\psi \rightarrow \varphi_i) \wedge (\neg\,\psi \rightarrow \varphi_j)) \mid i,j \in 1 \,..\, n\}$$

$\Phi = \mathbf{K}\,v \vee \mathbf{K}\,\neg\,v$ α = test (ψ) = test ($u \wedge v$)

$$\mathrm{Reg}_E(\Phi, \alpha) = \cancel{\mathbf{K}\,((u \wedge v \rightarrow v) \wedge ((\neg u \vee \neg v) \rightarrow v))}\ \mathbf{K}\,v$$
$$\vee\, \mathbf{K}\,((u \wedge v \rightarrow v) \wedge ((\neg u \vee \neg v) \rightarrow \neg v))$$
$$\vee\, \mathbf{K}\,((u \wedge v \rightarrow \neg v) \wedge ((\neg u \vee \neg v) \rightarrow v))$$
$$\vee\, \mathbf{K}\,((u \wedge v \rightarrow \neg v) \wedge ((\neg u \vee \neg v) \rightarrow \neg v))$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$Reg_E(\Phi, \alpha) = \bigvee \{\mathbf{K}\ ((\psi \rightarrow \varphi_i) \wedge (\neg \psi \rightarrow \varphi_j)) \mid i,j \in 1 .. n\}$$

$\Phi = \mathbf{K}\ v \vee \mathbf{K}\ \neg v$ $\alpha = test(\psi) = test(u \wedge v)$

$$
\begin{aligned}
Reg_E(\Phi, \alpha) = \ & \mathbf{K}\ v \\
& \vee \mathbf{K}\ ((u \wedge v \rightarrow v) \wedge ((\neg u \vee \neg v) \rightarrow \neg v)) \\
& \vee \mathbf{K}\ ((u \wedge v \rightarrow \neg v) \wedge ((\neg u \vee \neg v) \rightarrow v)) \\
& \vee \cancel{\mathbf{K}\ ((u \wedge v \rightarrow \neg v) \wedge ((\neg u \vee \neg v) \rightarrow \neg v))}\ \mathbf{K} \neg v
\end{aligned}
$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$\mathrm{Reg}_E\ (\Phi, \alpha) = \bigvee \left\{ \mathbf{K}\ ((\psi \rightarrow \varphi_i\ ) \wedge (\neg\ \psi \rightarrow \varphi_j\ ))\ \middle|\ i,j \in 1\ ..\ n \right\}$$

$\Phi = \mathbf{K}\ v \vee\ \mathbf{K} \neg\ v$ $\qquad$ $\alpha = \text{test}\ (\psi) = \text{test}\ (u \wedge v)$

$$\begin{aligned}
\mathrm{Reg}_E\ (\Phi, \alpha) = \quad & \mathbf{K}\ v \\
& \vee\ \mathbf{K}\ (\overset{\top}{\cancel{(u \wedge v \rightarrow v}}\ ) \wedge \overset{v \rightarrow u}{\cancel{((\neg u \vee \neg v) \rightarrow \neg v\ )}}\ ) \\
& \vee\ \mathbf{K}\ ((u \wedge v \rightarrow \neg v\ ) \wedge ((\neg u \vee \neg v) \rightarrow\ v\ )\ ) \\
& \vee\ \mathbf{K} \neg v
\end{aligned}$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\,\varphi_1 \vee \ldots \vee \mathbf{K}\,\varphi_n$ complex knowledge state

$$\mathrm{Reg}_E\,(\Phi, \alpha) = \bigvee \left\{ \mathbf{K}\,((\psi \to \varphi_i\,) \wedge (\neg\,\psi \to \varphi_j\,)) \,\middle|\, i,j \in 1 \,..\, n \right\}$$

$\Phi = \mathbf{K}\,v \vee \mathbf{K}\,\neg\,v$ α = test (ψ) = test ($u \wedge v$)

$$\begin{aligned}\mathrm{Reg}_E\,(\Phi, \alpha) = \quad & \mathbf{K}\,v \\ \vee\; & \mathbf{K}\,(v \to u) \\ \vee\; & \mathbf{K}\,(\underset{u \,\to\, \neg v}{\cancel{(u \wedge v \to \neg v\,)}} \wedge \overset{v}{\cancel{((\neg u \vee \neg v) \to v\,)}}\,) \\ \vee\; & \mathbf{K}\neg v\end{aligned}$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$\mathrm{Reg}_E\,(\Phi, \alpha) = \bigvee \{\mathbf{K}\,((\psi \to \varphi_i) \wedge (\neg\,\psi \to \varphi_j)) \mid i,j \in 1\,..\,n\}$$

$\Phi = \mathbf{K}\ v \vee \mathbf{K}\ \neg\, v$ $\alpha = \text{test}\,(\psi) = \text{test}\,(u \wedge v)$

$$\begin{aligned}\mathrm{Reg}_E\,(\Phi, \alpha) = \quad & \mathbf{K}\ v \\ \vee\ & \mathbf{K}\ (v \to u) \\ \vee\ & \mathbf{K}\ (v \wedge \neg\, u) \\ \vee\ & \mathbf{K} \neg v\end{aligned}$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\ \varphi_1 \vee \ldots \vee \mathbf{K}\ \varphi_n$ complex knowledge state

$$\mathrm{Reg}_E\,(\Phi, \alpha) = \bigvee \left\{ \mathbf{K}\,((\psi \rightarrow \varphi_i) \wedge (\neg\, \psi \rightarrow \varphi_j)) \,\middle|\, i,j \in 1 \,..\, n \right\}$$

$\Phi = \mathbf{K}\ v \vee \mathbf{K}\ \neg\ v$ $\qquad\alpha = \mathrm{test}\ (\psi) = \mathrm{test}\ (u \wedge v)$

$$\begin{aligned}\mathrm{Reg}_E\,(\Phi, \alpha) = \quad & \mathbf{K}\ v \\ & \vee\ \mathbf{K}\ (v \rightarrow u) \\ & \vee\ \cancel{\mathbf{K}\ (v \wedge \neg\, u)} \\ & \vee\ \cancel{\mathbf{K} \neg v}\end{aligned}$$

Epistemic actions in S5 : regression

$\Phi = \mathbf{K}\, \varphi_1 \vee \ldots \vee \mathbf{K}\, \varphi_n$ complex knowledge state

$$Reg_E(\Phi, \alpha) = \bigvee \{\mathbf{K}\, ((\psi \to \varphi_i) \wedge (\neg \psi \to \varphi_j)) \mid i,j \in 1 .. n\}$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v$ α = test (ψ) = test ($u \wedge v$)

$$\boxed{Reg_E(\Phi, \alpha) = \mathbf{K}\, v \vee \mathbf{K}\, (v \to u)}$$

Plans

- λ (empty plan) is a plan
- for any action $\alpha \in A_O \cup A_E$, $\langle\alpha\rangle$ is a plan
- if π and π' are plans then $\pi\,;\,\pi'$ is a plan
- if π and π' are plans and Φ a complex epistemic state then

  if Φ then π else π'

  is a plan

Progression of a CKS by a plan

- $\mathbf{Prog}(\Phi, \lambda) = \Phi$
- $\mathbf{Prog}(\Phi, \langle\alpha\rangle) = \mathbf{prog}(\Phi, \alpha)$
- $\mathbf{Prog}(\Phi, (\pi ; \pi')) = \mathbf{Prog}(\mathbf{Prog}(\pi, \Phi), \pi')$
- $\mathbf{Prog}(\Phi, \textbf{if } \Psi \textbf{ then } \pi \textbf{ else } \pi') = \begin{cases} \mathbf{Prog}(\Phi, \pi) & \text{si } \Phi \models \Psi \\ \mathbf{Prog}(\Phi, \pi') & \text{sinon} \end{cases}$

Planning problem

$$\mathcal{P} = \langle\, \mathbf{K}\, \varphi_{init}\ , A_O \cup A_E\ , \Gamma \,\rangle$$

simple knowledge state   goal = complex knowledge state

π is a valid plan for $\mathcal{P}$ iff Prog $(\Phi, \pi) \models \Gamma$

Plan verification : example

$$P = \langle\, \mathbf{K}\, \varphi_{init}\ ,\ A_O \cup A_E\ ,\ \Gamma\, \rangle$$

$$\begin{cases} \varphi_{init} = \top & \\ A_O = \{\gamma\} & \gamma = \text{switch}\,(u) \\ A_E = \{\alpha, \beta\} & \alpha = \text{test}\,(u \wedge v) \qquad \beta = \text{test}\,(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K}\, \neg v & \end{cases}$$

Plan verification : example

$$P = \langle \mathbf{K}\, \varphi_{init} , A_O \cup A_E , \Gamma \rangle$$

$$\begin{cases} \varphi_{init} = \top & \\ A_O = \{\gamma\} & \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} & \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v & \end{cases}$$

$$\pi = \beta;\ \text{if } \mathbf{K}\,(u \leftrightarrow v) \text{ then } \beta \text{ else } (\gamma; \alpha)$$

Plan verification : example

$$P = \langle \mathbf{K}\, \varphi_{init} , A_O \cup A_E , \Gamma \rangle$$

$$\begin{cases} \varphi_{init} = \top & \\ A_O = \{\gamma\} & \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} & \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v & \end{cases}$$

$$\pi = \beta;\ \text{if } \mathbf{K}\,(u \leftrightarrow v) \text{ then } \alpha \text{ else } (\gamma\ ;\ \alpha)$$

$$\text{Prog}\,(\mathbf{K}\top, \pi) = \mathbf{K}\,(u \wedge v) \wedge \mathbf{K}\,(\neg u \wedge \neg v)$$

Plan verification : example

$$P = \langle \mathbf{K}\, \varphi_{init}\ ,\ A_O \cup A_E\ ,\ \Gamma \rangle$$

$$\begin{cases} \varphi_{init} = \top & \\ A_O = \{\gamma\} & \gamma = \text{switch}\ (u) \\ A_E = \{\alpha, \beta\} & \alpha = \text{test}\ (u \wedge v) \qquad \beta = \text{test}\ (u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v & \end{cases}$$

$$\pi = \beta; \text{ if } \mathbf{K}\ (u \leftrightarrow v) \text{ then } \alpha \text{ else } \gamma\ ;\ \alpha$$

$$\text{Prog}\ (\mathbf{K}\top, \pi) = \mathbf{K}\ (u \wedge v) \wedge \mathbf{K}\ (\neg u \wedge \neg v) \models \Gamma$$

π valid for P

Plan generation

$$P = \langle \mathbf{K}\,\varphi_{init}\ ,\ A_O \cup A_E\ ,\ \Gamma = \mathbf{K}\,G_1 \vee \ldots \vee \mathbf{K}\,G_n \rangle$$

Φ **:=** Γ ;

repeat

choose an action $\alpha \in A_O \cup A_E$;

Φ **:=** $\mathrm{Reg}\ (\Phi, \alpha\ ) \vee \Phi$

until $\mathbf{K}\,\varphi_{init} \models \Phi$ or Φ can no longer be changed

Plan generation : example

$$P = \langle \mathbf{K}\,\varphi_{init}\,,\, A_O \cup A_O\,,\, \Gamma = \mathbf{K}\,G_1 \vee \ldots \vee \mathbf{K}\,G_n \rangle$$

$\Phi := \Gamma$;

$\sigma := \{ \langle \Gamma, \lambda \rangle \}$

repeat

choose an action $\alpha \in A_O \cup A_E$;

$\Phi := \mathrm{Reg}\,(\Phi, \alpha\,) \vee \Phi$

$\sigma := \sigma \cup \{ \langle \mathrm{Reg}\,(\Phi, \alpha\,) \wedge \neg\,\Phi, \alpha \rangle \}$

until $\mathbf{K}\,\varphi_{init} \models \Phi$ or Φ can no longer be changed

Plan generation : example

$P = \langle \mathbf{K}\, \varphi_{init}\ , A_O \cup A_O\ , \Gamma = \mathbf{K}\, G_1 \vee \ldots \vee \mathbf{K}\, G_n \rangle$

$\Phi := \Gamma\ ;$

$\sigma := \{ \langle \Gamma, \lambda \rangle \}$

repeat

choose an action $\alpha \in A_O \cup A_E$;

$\Phi := Reg\ (\Phi, \alpha\ ) \vee \Phi$

$\sigma := \sigma \cup \{ \langle Reg\ (\Phi, \alpha\ )\ , \alpha \rangle \}$

until $\mathbf{K}\, \varphi_{init} \models \Phi$ or Φ can no longer be changed

Plan generation : example

$P = \langle \mathbf{K}\ \varphi_{init}\ , A_O \cup A_O\ , \Gamma = \mathbf{K}\ G_1 \vee \ldots \vee \mathbf{K}\ G_n \rangle$

$\Phi := \Gamma\ ;$

$\sigma := \{ \langle \mathbf{K}\ G_1\ , \lambda \rangle, \ldots , \langle \mathbf{K}\ G_n\ , \lambda \rangle \}$

repeat

choose an action $\alpha \in A_O \cup A_E$;

$\Phi := \mathrm{Reg}\ (\Phi, \alpha\ ) \vee \Phi$

$\sigma := \sigma \cup \{ \langle \mathbf{K}\ \varphi_i\ , \alpha \rangle \mid \mathrm{Reg}\ (\Phi, \alpha\ ) \models \varphi_i\ ,$

$\nexists \langle \mathbf{K}\ F_j\ , \alpha \rangle$ in σ s.t. $F_j \models \varphi_i \}$

until $\mathbf{K}\ \varphi_{init} \models \Phi$ or Φ can no longer be changed

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v \qquad \sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle \}$$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v \qquad \sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle \}$$

Regression by α $\qquad$ $\text{Reg}(\Phi, \alpha) = \mathbf{K}\, v \vee \mathbf{K}(v \rightarrow u)$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v \qquad \sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle \}$$

Regression by α $\qquad \mathrm{Reg}(\Phi, \alpha) = \mathbf{K}\, v \vee \mathbf{K}\,(v \rightarrow u)$

$$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v \vee \mathbf{K}\,(v \rightarrow u)$$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = switch\ (u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = test\ (u \wedge v) \qquad \beta = test\ (u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v$ $\qquad \sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle \}$

Regression by α $\qquad$ $Reg\ (\Phi, \alpha) = \mathbf{K}\, v \vee \mathbf{K}\, (v \to u)$

$\cancel{\Phi = \mathbf{K}\, v \vee \mathbf{K} \neg v \vee \mathbf{K}\, (v \to u)}$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}\, (v \to u)$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}\,(v \to u)$ $\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle$
$\langle \mathbf{K}\,(v \to u), \alpha \rangle \}$

Regression by α $\mathrm{Reg}(\Phi, \alpha) = \mathbf{K}\, v \vee \mathbf{K}\,(v \to u)$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}\,(v \to u)$

$$
\begin{cases}
\varphi_{init} = \top \\
A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\
A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\
\Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v
\end{cases}
$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}(v \rightarrow u)$

$\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle$
$\langle \mathbf{K}(v \rightarrow u), \alpha \rangle,$
$\langle \mathbf{K}(v \rightarrow \neg u), \gamma \rangle \}$

Regression by γ

$\text{Reg}(\Phi, \gamma) = \mathbf{K}\, v \vee \mathbf{K}(v \rightarrow \neg u)$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}(v \rightarrow u) \vee \mathbf{K}(v \rightarrow \neg u)$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}(u \wedge v) \qquad \beta = \text{test}(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v \end{cases}$$

$\Phi = \mathbf{K}\, v \vee \mathbf{K}\,(v \rightarrow u)$

$\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle$
$\langle \mathbf{K}\,(v \rightarrow u), \alpha \rangle,$
$\langle \mathbf{K}\,(v \rightarrow \neg u), \gamma \rangle,$
$\langle \mathbf{K} \top, \beta \rangle \}$

Regression by β

$\text{Reg}(\Phi, \beta) = \mathbf{K} \top$

$\Phi = \mathbf{K} \top = \mathbf{K}\, \varphi_{init}$ STOP

$$
\begin{cases}
\varphi_{init} = \top \\
A_O = \{\gamma\} \qquad \gamma = \text{switch}\ (u) \\
A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}\ (u \wedge v) \qquad \beta = \text{test}\ (u \leftrightarrow v) \\
\Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v
\end{cases}
$$

$$
\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle, \langle \mathbf{K}\, (v \rightarrow u), \alpha \rangle, \langle \mathbf{K}\, (v \rightarrow \neg u), \gamma \rangle, \langle \mathbf{K} \top, \beta \rangle \}
$$

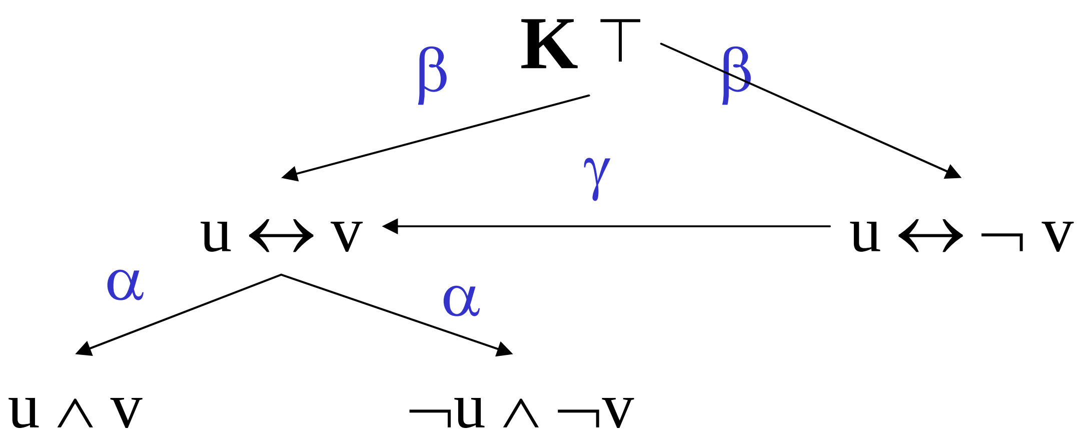
$$
\begin{cases}
\varphi_{init} = \top \\
A_O = \{\gamma\} \qquad \gamma = \text{switch}\,(u) \\
A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}\,(u \wedge v) \qquad \beta = \text{test}\,(u \leftrightarrow v) \\
\Gamma = \mathbf{K}\, v \vee \mathbf{K} \neg v
\end{cases}
$$

$$
\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K} \neg v, \lambda \rangle, \langle \mathbf{K}\, (v \rightarrow u), \alpha \rangle, \\
\langle \mathbf{K}\, (v \rightarrow \neg u), \gamma \rangle, \langle \mathbf{K} \top, \beta \rangle \}
$$

$$
\begin{tikzcd}
 & & \mathbf{K} \top \arrow[dl, "\beta"'] \arrow[dr, "\beta"] & \\
 & u \leftrightarrow v \arrow[dl, "\alpha"'] \arrow[dr, "\alpha"] & & u \leftrightarrow \neg v \arrow[ll, "\gamma"'] \\
u \wedge v & & \neg u \wedge \neg v &
\end{tikzcd}
$$

$$\begin{cases} \varphi_{init} = \top \\ A_O = \{\gamma\} \qquad \gamma = \text{switch}\,(u) \\ A_E = \{\alpha, \beta\} \qquad \alpha = \text{test}\,(u \wedge v) \qquad \beta = \text{test}\,(u \leftrightarrow v) \\ \Gamma = \mathbf{K}\, v \vee \mathbf{K}\, \neg v \end{cases}$$

$$\sigma = \{ \langle \mathbf{K}\, v, \lambda \rangle, \langle \mathbf{K}\, \neg v, \lambda \rangle, \langle \mathbf{K}\, (v \rightarrow u), \alpha \rangle, \langle \mathbf{K}\, (v \rightarrow \neg u), \gamma \rangle, \langle \mathbf{K} \top, \beta \rangle \}$$



π : β;
if $\mathbf{K}\,(u \leftrightarrow v)$
then α
else γ; α

knowledge-based programs

(Fagin-Halpern-Moses-Vardi, 95)

involve deduction tasks at execution time

π : β;

if $\mathbf{K}$ $(u \leftrightarrow v)$

then α

else γ; α

Bibliography

• logical representation of ontic actions

- E. Sandewall, *Features and fluents*, Oxford University Press,1995
- F. Giunchiglia, J. Lee, V. Lifschitz, N. McCain & H. Turner, Nonmonotonic causal theories, *Artificial Intelligence Journal* 153, 2004
- R. Reiter, *Knowledge in Action*, MIT Press, 2001;
- F. Lin, From causal theories to successor state axioms and STRIPS-like systems, *AAAI-00*
- J. Lang, F. Lin & P. Marquis, Causal theories of action: a computational core, *IJCAI-03*

Bibliography

- **logical modelling of sensing actions**

- H. Levesque, What is planning in the presence of sensing? *AAAI-96*
- R. Scherl and H. Levesque. Knowledge, action, and the frame problem. *Artificial Intelligence Journal* 144, 2003
- C. Baral & T. Son, Formalizing sensing actions – A transition-bsed approach. *Artificial Intelligence Journal* 125, 2001
- G. de Giacomo and R. Rosati, Minimal knowledge approach to reasoning about action and sensing, *Electronic Transactions on Artificial Intelligence*, 1999
- A. Herzig, J. Lang, D. Longin and T. Polacsek. A logic for planning under partial observability. *AAAI-00*.

Bibliography

- **logic-based partially observable planning**

  - P. Bertoli, A. Cimatti, M. Roveri and P. Traverso, Planning in nondeterministic domains under partial observability via symbolic model checking, *IJCAI-01*

  - J. Rintanen, Backward plan construction for planning under partial observability, *AIPS-02*

  - A. Herzig, J. Lang & P. Marquis, Action representation, progression and partially observable planning in epistemic logic, *IJCAI-03*

- **knowledge-based programs**

  - R. Fagin, J. Halpern, Y. Moses and M. Vardi. *Reasoning about knowledge*, MIT Press, 1995